9

Quadratic Equation & Linear Inequalities

- **Quadratic Equation**

The equation in the form of $ax^2 + bx + c = 0$, $a \neq 0$

For example $5x^2 + 9x + 7 = 0$ is quadratic equation.

- **Roots of a Quadratic Equation**

➢ The value of the variables for which equation is satisfied is known as roots of the quadratic equation.

➢ In a quadratic equation, it has two roots.

- **Solving Quadratic equation**

(i) Factorization Method

By splitting the middle term and taking the common factors.

If $(x - \alpha)$ and $(x - \beta)$ be the two factors of a quadratic equal $ax^2 + bx + c = 0$, then $x = \alpha, \beta$ be the two roots

(ii) Quadratic Formula

In $ax^2 + bx + c = 0$, $a \neq 0$ the roots are

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$D = b^2 - 4ac$ is called as Discriminant

(i) If $D > 0$, then equation have two real and distinct roots.

(ii) If $D = 0$, then equation have two real and equal roots.

(iii) If $D < 0$, then equation have no real roots. It will have imaginary complex roots.

- **Relation Between Roots and Co-efficient of Quadratic Equation**

If $\propto$, β are roots of the quadratic equation, then

(i) $\alpha + \beta = \frac{-b}{a}$

(ii) $\alpha\beta = \frac{c}{a}$

- **Inequalities**

A statement involving a sign of inequality as: $>, <, \geq, \leq$ is called as inequalities

For example: $2x + 3 > 5$

$$3x + a \leq 7$$

- **Solving of Inequalities (Rules)**

(1) Equal numbers may be added or subtracted from both side of inequalities.
   (i) If $a > b$, then $a + x > b + x$ and $a - x > b - x$
   (ii) If $a \leq b$, then $a + x \leq b + x$ and $a - x \leq b - x$

(2) Both side of an inequalities, can be multiplied and divided by same positive number.
   (i) If $a > b$, then $ax > bx$, and $\frac{a}{x} > \frac{b}{x}$
   (ii) If $a \leq b$, then $ax \leq bx$, and $\frac{a}{x} > bx$

(3) When both sides of inequalities are multiplied by same negative number, then sign or inequality gets reversed.

   (i) If $a > b$, and $x < 0$, then $ax < bx$, $\frac{a}{x} < \frac{b}{x}$
   (ii) $a \leq b$, and $x \leq 0$, and $ax \geq bx$, $\frac{a}{x} \geq \frac{b}{x}$

Check Your Progress

1. If $X^2+bx+c = 0$ and $x^2 + cx + b = 0$ have exactly one common root then what is the value of (c+b)?
   A. 0
   B. 1
   C. -1
   D. None of the above
2. If α and β are the roots of $4x^2-6x - 12=0$, then what is the equation whose roots are $\alpha^2 + 2$ and $\beta^2 + 2$?
   A. $8x^2 + 98x + 236 = 0$
   B. $8x^2 - 98x + 236 = 0$
   C. $8x^2 - 98x - 236 = 0$
   D. $x^2 - 98x + 236 = 0$
3. What will be the product of x*z if the equation $y^2+xy+z=0$ and $y^2 +4y+3=0$ have one common root?
   A. 12
   B. -12
   C. 7
   D. -7
4. If α and β are the roots of the quadratic equation $5x^2 - 15x + 20 = 0$ .Value of $\alpha^2 + \beta^2$
   A. 1
   B. -1
   C. 0
   D. 2
5. Solution of $x^2 + 10$ i x -21 =0 are
   A. -3 i, 7i
   B. -3i,-7i
   C. 3i,7i
   D. 3 i, -7 i
6. By solving the inequality ½(4x+3) > 1/3(x+4), the answer will be
   A. x > - 1/10
   B. x > 1/10
   C. x > 1/5
   D. x > -1/5
7. By solving the inequality 10 a – 4 > 8 ,the value of a is
   A. Greater than 2
   B. Less than 2
   C. Equal to 2
   D. Less than 1
8. The imaginary roots of the equation $(x^2 + 2)^2 + 8x^2 = 6x(x^2 + 2)$ are -

(A) $1 \pm i$ (B) $2 \pm$
(C) $-1 \pm i$ (D) None of these

9. Both roots of the equation $(x - b)(x - c) + (x - c)(x - a) + (x - a)(x - b) = 0$ are -
   (A) positive (B) negative
   (C) real (D) imaginary
10. If p and q are roots of the equation $x^2 - 2x + A = 0$ and r and s be roots of the equation $x^2 - 18x + B = 0$ if $p < q < r < s$ be in A.P., then A and B are respectively-
   (A) – 3, 77 (B) 3, 77
   (C) 3,– 77 (D) None of these
11. Both roots of the equation $(x - b)(x - c) + (x - c)(x - a) + (x - a)(x - b) = 0$ are -
   (A) positive
   (B) negative
   (C) real
   (D) imaginary
12. If x is real then the value of the expression $\frac{x^2 + 14x + 9}{x^2 + 2x + 3}$ lies between
   (A) –3 and 3
   (B) –4 and 5
   (C) –4 and 4
   (D) –5 and 4
13. If the roots of the equations $x^2 + 3x + 2 = 0$ and $x^2 - x + p = 0$ are in the same ratio then the value of p is given by-
   (A) 2/7 (B) 2/9
   (C) 9/2 (D) 7/2
14. The sum of all real roots of the equation
   $|x - 2|^2 + |x - 2| - 2 = 0$, is-
   (A) 0 (B) 8
   (C) 4 (D) None of these
15. If roots of the equation $x^2 + ax + 25 = 0$ are in the ratio of 2 : 3 then the value of a is -
   (A) $\frac{\pm 5}{\sqrt{6}}$ (B) $\frac{\pm 25}{\sqrt{6}}$
   (C) $\frac{\pm 5}{6}$ (D) None of these

Answer to check your progress

1 B 2 B 3 A 4 A 5 B 6A 7 B 8A 9C 10 A 11C 12D 13 B 14C 15 B

Stretch Yourself

1. Find value of k if $x^2 + k(2x + 3) + 4(x + 2) + 3k - 5$ is a perfect square
2. Find the solution of the equation $2x^2 + 3x - 9 = 0$
3. If $x + 1$ is a factor of the expression
4. $x^4 + (p - 3)x^3 - (3p - 5)x^2 + (2p - 9)x + 6$ then find the value of p

5. If $x^2 + 2xy + 2x + my - 3$ have two rational factors then find m.
6. Find the nature of roots of the equation
7. $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$