4

TRIGONOMETRIC FUNCTIONS-II

Addition and Multiplication of Trigonometric Functions

Addition Formulae

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A + \tan B}{1 + \tan A \tan B}$$

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$\cot(A-B) = \frac{\cot A \cot B + 1}{\cot A - \cot B}$$

$$\tan\left(\frac{\pi}{4} + A\right) = \frac{1 + \tan A}{1 - \tan A}$$

TRANSFORMATION OF PRODUCTS INTO SUMS AND VICE VERSA

Transformation of Products into Sums or Differences

$$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$$

$$\sin(A+B) - \sin(A-B) = 2 \cos A \sin B$$

$$\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$$

$$\cos(A-B) - \cos(A+B) = 2 \sin A \sin B$$

Transformation of Sums or Differences into Products

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos D - \cos C = 2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

Further Applications of Addition and Subtraction Formulae

$$\sin(A+B)\sin(A-B) = sin^2 A - sin^2 B$$

$$\cos(A+B)\cos(A-B) = cos^2 A \text{ -} sin^2 B$$

Trigonometric Functions of Multiples of Angles

(a) To express sin 2A in terms of sin A, cos A and tan A.

$$\sin 2A = 2 \sin A \cos A$$

$$\sin 2A = \frac{2 \sin A \cos A}{sin^2 A + cos^2 A}$$

$$\sin 2A = \frac{2tanA}{1 + tan^2 A}$$

(b) To express cos 2A in terms of sin A, cos A and tan A.

$$\cos 2A = cos^2 A - sin^2 A$$

$$\cos 2A = 2cos^2A - 1$$

$$\cos 2A = 1 - 2sin^2A$$

$$\cos 2A = \frac{1 - tan^2A}{1 + tan^2A}$$

(c) To express tan 2A in terms of tan A

$$\tan 2A = \frac{2tanA}{1 - tan^2A}$$

(d) Trigonometric Functions of 3A in Terms of A

(a) sin 3A in terms of sin A

$$\sin 3A = 3\sin A - 4sin^3A$$

(b) cos 3A in terms of cos A

$$\mathbf{\cos 3A = 4cos^3A - 3\cos A}$$

(c) tan 3A in terms of tan A

$$\mathbf{\tan 3A = \frac{3tanA - tan^3A}{1 - 3tan^2A}}$$

Trigonometric Functions of Submultiples of Angles

A/2,A/3,A/4are called submultiples of A

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+cosA}{2}}$$

$$\tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

sinθ= sin α general solution of the equation

$$\theta = n\pi + (-1)^n\alpha, n\epsilon Z$$

cos θ= cos α general solution of the equation

$$\theta = 2n\pi \pm \alpha, n\epsilon Z$$

tan θ= tan α general solution of the equation

$$\theta = n\pi + \alpha, n\epsilon Z$$

cosec θ= cosec α general solution of the equation

$$\theta = n\pi + (-1)^n\alpha$$

sec θ=sec α general solution of the equation

$$\theta = 2n\pi \pm \alpha$$

cot θ= cotα general solution of the equation

$$\theta = n\pi + \alpha$$

Check Your Progress

1 The maximum value of $5\cos\theta + 12\sin\theta$ is equal to:
(A) 13
(B) 17
(C) 12
(D) 5

2 If $\sin A = \frac{1}{\sqrt{10}}, \sin B = \frac{1}{\sqrt{5}}$ then the value of A+B is equal to:
(A) $\frac{\pi}{2}$
(B) π
(C) $\frac{\pi}{4}$
(D) 2π

3 What is the general solution of the equation $\sin\theta = \sin\alpha$?
(A) $\theta = n\pi + (-1)\alpha$
(B) $\theta = n\pi + \alpha$
(C) $\theta = 2n\pi \pm \alpha$
(D) $\theta = n\pi + (-1)^n\alpha$

4 If $\sin A\frac{8}{17}$ and $\sin B\frac{5}{13}$, then $\sin(A-B)$ is equal to:
(A) $\frac{21}{221}$
(B) $\frac{112}{121}$
(C) $\frac{13}{17}$
(D) $\frac{169}{25}$

5 The value of $\tan 15^\circ$ is equal to:

(A) $\sqrt{3}+1$
(B) $\sqrt{3}-1$
(C) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$
(D) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

Stretch yourself

Q 1 Prove that $\frac{1+\sin 2\theta+\cot 2\theta}{1+\sin 2\theta-\cos 2\theta}=\cot\theta$

Q2 Prove that $\sin 12^o \cdot \sin 48^o \cdot \sin 54^o = \frac{1}{8}$

Q 3 If three angles A,B,C are in Arithmetic Progression (AP) prove that

$\cot B = \frac{\sin A - \sin C}{\cos C - \cos A}$

Q4 Express the following as sum or difference

(i) $2\sin 3\theta \cdot \cos\theta$
(ii) $\cos 3\theta \cdot \cos 5\theta$

Q 5If $2\cos\theta = 1$, then find its general solution.

Answer to Check Your Progress

Q 1 A

Q 2 C

Q 3 D

Q 4 A

Q 5 C

Answer to Stretch yourself

Q 1 Prove by using trigonometric function of multiple sum and difference of two angle.

Q 2 Prove by value of trigonometric function of some special angle.

Q3 Here A,B, C are in A.P

than 2B=A+C or $B=\frac{A+C}{2}$

Q 4 (i) $\sin 4\theta + \sin 2\theta$

$\frac{1}{2}[\cos 8\theta + \cos 2\theta]$

Q 5 $\theta = 2n\pi \pm \frac{\pi}{3}, n \in z$