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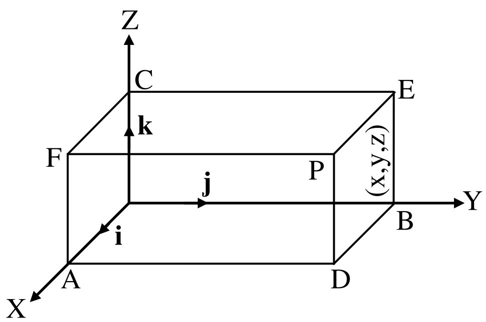
INTRODUCTION TO THREE DIMENSIONAL GEOMETRY

Let O be a fixed point known as origin and let OX, OY and OZ be three mutually perpendicular lines, taken as x-axis, y-axis and z-axis respectively in such a way that they form a right - handed system.



The planes XOY, YOZ and ZOX are known as xy-plane, yz-plane and zx-plane respectively.

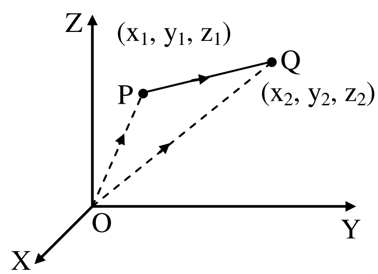
Let P be a point in space and distances of P from yz, zx and xy-planes be x,y,z respectively (with proper signs), then we say that coordinates of P are (x, y, z). Also OA = x, OB = y, OC = z.

Distance Formula

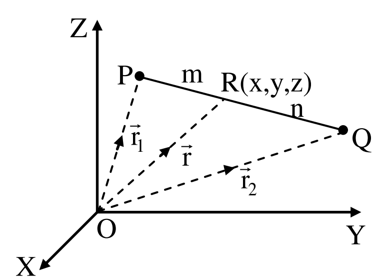
If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are two points, then distance between them $PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

In particular distance of a point (x, y, z) from origin $= \sqrt{x^2 + y^2 + z^2}$.

Section Formula



Coordinates of the point dividing the line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in the ratio $m_1 : m_2$ are



(i) In case of internal division

$$\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2},\frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$$

(ii) In case of external division

$$\left(\frac{m_1x_2-m_2x_1}{m_1-m_2},\frac{m_1y_2-m_2y_1}{m_1-m_2},\frac{m_1z_2-m_2z_1}{m_1-m_2}\right)$$

Coordinates of the Mid point :

When division point is the midpoint of PQ, then ratio will be 1 : 1; hence coordinates of the midpoint of PQ are $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$

Centroid of a Triangle :

If (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) be the vertices of a triangle, then the centroid of the triangle is

$$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$$

Check Your Progress

1. The points A(1, –1, – 5), B(3, 1, 3) and C(9, 1, –3) are the vertices of-

   (A) an equilateral triangle

   (B) an isosceles triangle

   (C) a right angled triangle

   (D) none of these

2. Distance of the point (x, y, z) from y-axis is-

   (A) y (B) $\sqrt{x^2+y^2}$

   (C) $\sqrt{y^2+z^2}$ (D) $\sqrt{z^2+x^2}$

3. The distance of a point P(x, y, z) from yz plane is-

   (A) x (B) y

   (C) z (D) x + y + z

4. A point which lie in yz plane, the sum of co-ordinate is 3, if distance of point from xz plane is twice the distance of point from xy plane, then co-ordinates are-

   (A) (1, 2, 0) (B) (0, 1, 2)

   (C) (0, 2, 1) (D) (2, 0, 1)

5. A point located in space is moves in such a way that sum of distance from xy and yz plane is equal to distance from zx plane the locus of the point are-

   (A) $x - y + z = 2$ (B) $x + y - z = 0$

   (C) $x + y - z = 2$ (D) $x - y + z = 0$

6. A (1, 3, 5) and B (– 2, 3, – 4) are two points, A point P moves such that $PA^2 - PB^2 = 6c$, then locus of P is-

   (A) x + 3z + 1 – c = 0

   (B) x + 3z – 1 + c = 0

   (C) 2x + 3z + 1 – c = 0

   (D) 2x + 3z – 1 + c = 0

7. Find the ratio in which the segment joining
   (1, 2, –1) and (4, –5, 2) is divided by the plane 2x – 3y + z = 4.

   (A) 2 : 1 (B) 3 : 2

   (C) 3 : 7 (D) 1 : 2

8. If points A (3, 2, –4); B(5,4, –6) and C(9, 8,–10) are collinear then B divides AC in the ratio-

   (A) 2 : 1 (B) 1 : 2

   (C) 2 : 3 (D) 3 : 2

9. If zx plane divides the line joining the points
   (1, –1, 5) and (2, 3, 4) in the ratio **m**:1 then **m**equals to-

   (A) 1/3 (B) 3

   (C) –3 (D) –1/3

10. OABC is a tetrahedron whose vertices are
    O (0, 0, 0); A (a, 2, 3); B (1, b, 2) and C (2, 1, c) if its centroid is (1, 2, –1) then distance of point (a, b, c) from origin are-

    (A) $\sqrt{14}$ (B) $\sqrt{107}$

    (C) $\sqrt{107/14}$ (D) None of these

11. The ratio in which the yz-plane divides the join of the points (–2, 4, 7) and (3, –5, 8) is-

    (A) 2 : 3 (B) 3 : 2

    (C) –2 : 3 (D) 4 : –3

12. A (3, 2, 0), B (5, 3, 2) and C (–9, 6, –3) are vertices of a triangle ABC. If the bisector of ∠A meets BC at D, then its coordinates are-

    (A) $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$ (B) $\left(-\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$

    (C) $\left(\frac{19}{8}, \frac{57}{16}, -\frac{17}{16}\right)$ (D $\left(-\frac{19}{8}, -\frac{57}{16}, \frac{17}{16}\right)$

13. If origin is the centroid of the triangle ABC with vertices A(a, 1, 3), B(–2, b, –5) and C(4, 7, c) then values of a, b, c are respectively-

(A) 2, 8, 2 (B) 0, 2, 2

(C) –2, –8, 2 (D) None of these

14. The line joining the points (2, –3, 1) and (3, –4, –5) and cuts the plane 2x + y + z =7 in those points, the point are-

(A) (1, 2, 7) (B) (–1, 2, 7)

(C) (1, –2, 7) (D) (1, –2, –7)

15. The vertices of a triangle ABC are A (4, 3, –2), B(3, 0, 1) and C(2, –1 , 3), the length of the median drawn from point 'A' -

(A) $\frac{1}{2}\sqrt{122}$ (B) $\sqrt{122}$

(C) $\frac{1}{3}\sqrt{122}$ (D) None of these

Hint to Check Your Progress

1 A 2D 3A 4C 5D

6B 7C 8B 9A 10 B

11A 12A 13C 14C 15A