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DETERMINANTS

An expression expressed in equal number of rows and column and put between two vertical lines is named as determinant of that expression

DETERMINANT OF ORDER 2

$a_1x+b_1y = c_1$

$a_2x+b_2y = c_2$

$$x = \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1}$$

$$y = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$$

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

The number $a_1 b_2 - a_2 b_1$ determines whether the values of x and y exist or not.

DETERMINANT OF ORDER 3

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} =$$

$$a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$$

Minor

The Determinant that is left by cancelling the row and column intersecting at a particular element is called the minor of that element.

If $\triangle = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ then Minor of a_{11} is

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}, \text{ Similarly}$$

$$M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

Using this concept the value of Determinant can be

$$\triangle = a_{11} M_{11} - a_{12} M_{12} + a_{13} M_{13}$$

Cofactor

The cofactor of an element a_{ij} is denoted by F_{ij} and is equal to $(-1)^{i+j} M_{ij}$ where M is a minor of element a_{ij}

if $\triangle = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

then $F_{11} = (-1)^{1+1} M_{11} = M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

$$F_{12} = (-1)^{1+2} M_{12} =$$

$$-M_{12} = -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

Property of Determinant

Property **-1**

The value of Determinant remains unchanged, if the rows and the column are interchanged. This is always denoted by ' and is also called transpose

Property **-2**

If any two rows (or columns) of a determinant be interchanged, the determinant is unaltered in numerical Value, but is changed in sign only,

Property **-3**

If a Determinant has two rows (or columns) identical, then its value is zero.

Property **-4**

If all the elements of any row (or column) be multiplied by the same number, then the value of Determinant is multiplied by that number.

Property **-5**

If each element of any row (or column) can be expressed as a sum of two terms, then the determinant can be expressed as the sum of the Determinants

Property **-6**

The value of a Determinant is not altered by adding to the elements of any row (or column) the same multiples of the corresponding elements of any other row (or column)

Property **-7**

If $\triangle = f(x)$ and $f(a) = 0$ then $(x-a)$ is a factor of $\triangle$

Application of Determinants

Area of Triangle

Area of a triangle ABC, (say) whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

Area of $(\triangle ABC) =$

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Condition of collinearity of three points

Let $A(x_1,y_1)$,$B(x_2,y_2)$ and $C(x_3,y_3)$ be three point then A ,B,C are called collinear if

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Equation of a line passing through the given two points

Let $A(x_1,y_1)$,$B(x_2,y_2)$ and $C(x,y)$ be any point on the line joining A and B .Then equation

$$\begin{vmatrix} x & y & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Check Your Progress

1. $\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$ is equal to -

   (A) $a^2b^2c^2$ (B) $2a^2b^2c^2$

   (C) $4a^2b^2c^2$ (D) None of these

2. If $\begin{vmatrix} 3^2+k & 4^2 & 3^2+3+k \\ 4^2+k & 5^2 & 4^2+4+k \\ 5^2+k & 6^2 & 5^2+5+k \end{vmatrix} = 0$, then the value of k is –

   (A) 2 (B) 1 (C) –1 (D) 0

3. If $\begin{vmatrix} a+1 & 1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix} = 4$, then the value a is -

   (A) 1 (B) –1 (C) –2 (D) 0

4. The value of $\begin{vmatrix} 5+i & -3i \\ 4i & 5-i \end{vmatrix}$ is -

   (A) 12 (B) 17

   (C) 14 (D) 24

5. $\begin{vmatrix} \sec x & \sin x & \tan x \\ 0 & 1 & 0 \\ \tan x & \cot x & \sec x \end{vmatrix}$ is equal to -

   (A) 0 (B) – 1

   (C) 1 (D) None of these

6. The cofactors of 1, –2, –3 and 4 in $\begin{vmatrix} 1 & -2 \\ -3 & 4 \end{vmatrix}$ are-

   (A) 4, 3, 2, 1 (B) –4, 3, 2, –1

   (C) 4, –3, –2, 1 (D) –4, –3, –2, –1

7. The minors of the elements of the first row in the determinant $\begin{vmatrix} 2 & -1 & 4 \\ 4 & 2 & -3 \\ 1 & 1 & 2 \end{vmatrix}$ are-

   (A) 2, 7, 11 (B) 7, 11, 2

   (C) 11, 2, 7 (D) 7, 2, 11

8. The value of the determinant $\begin{vmatrix} 1/a & 1 & bc \\ 1/b & 1 & ca \\ 1/c & 1 & ab \end{vmatrix}$ is equal to

   (A) abc (B) 1/abc

   (C) 0 (D) None of these

9. If $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$, then value of x are-

   (A) 0, a (B) 0, – a

   (C) a, – a (D) 0, 3a

10. $\begin{vmatrix} 7579 & 7589 \\ 7581 & 7591 \end{vmatrix} =$

   (A) 20 (B) – 2

   (C) – 20 (D) 4

Stretch Yourself

1. Find

$$\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix}$$

2. If $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = \lambda \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

   Than Find the value of λ

3. If $\triangle = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$ and $\triangle_2 = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$.

   Show that $\triangle_1$ is equal to $\triangle_2$

4. If $ax + by + cz = 1$, $bx + cy + az = 0 = cx$ + ay + bz, Find the value of $\begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix}$

   $\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

5. Calculate the value of $\begin{vmatrix} 441 & 442 & 443 \\ 445 & 446 & 447 \\ 449 & 450 & 451 \end{vmatrix}$

Hint Check Your Progress

1 C 2 B 3D 4 C 5 C

6 A 7 B 8 C 9 D 10 C