Learner Guide

211 - Mathematics

Secondary Course

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NATIONAL INSTITUTE OF OPEN SCHOOLING
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From the Director (Academic)....

Dear Learner,

Greetings!

It has been our motto to help the self learners attain their educational goals. The Learner Guide has been designed for the first time to help you learn better. The important points of the study materials have been highlighted in this guide and would give you a glimpse of the whole course at one go. It would assess you in revising the study material in a short time.

I feel this study guide, apart from deepening your understanding of the subject, will also help you in enhancing your performance in the examination.

I hope you will refer to it for revision and find it useful.

Best wishes for a bright future and prosperous life!

(Dr. Kuldeep Agarwal)
Director (Academic)

Message from Assistant Director

Dear Learner,

Now your problems will be solved in a click,

As NIOS brings the knowledge, at your finger tip!

Appreciating your need for support NIOS brings the magic of technology to your door step!! “Mukta Vidya Vani” our web based live PCPs supplement and complement the Self Learning Materials. It gives you an opportunity to interact with the experts of your subjects. You can clear your queries and doubts by calling on our TOLL Free Number **1800 180 2543.** You can also call on **0120-462649.** The time schedule of the live programmes is given below for reference. If due to any reason you miss the live PCPs you can hear the recorded versions in repeat cycle or at Audio on Demand.

We hope that you will access these ICT options for better understanding of content, concepts and clarification of your doubts. For listening to live or recorded PCPs on Mukta Vidya Vani, you can directly log on to www.nios.ac.in and click on Mukta Vidya Vani. You can also log on to http://www.nios.iradionindia.com/. NIOS also provides video programmes which are telecast through Doordarshan educational channel Gyandarshan and audio programme through Gyan Vani (FM) channel at 106.5 MHz.

ICT Option	Time Slots	
Mukta Vidya Vani	2.30 PM – 5.00 PM Monday–Friday 10.30AM – 12.30AM Saturday and Sunday	Recorded 24 x 7
DD 1	5.02 AM – 5.25 AM Every Friday	
Gyandarshan	6.30 PM – 7.00 PM Every day	
Gyan Vani (FM)	8.30 AM – 9.00 AM Every Friday, Saturday and Sunday	4.30 PM – 5.00 PM Repeat broadcast on Friday, Saturday and Sunday

We look forward to your greater participation and interaction!

Dr. Rachna Bhatia
Assistant Director (Academic)

Contents

INTRODUCTION

Mathematics is the base of human civilization. From cutting vegetables to arranging books on the shelves, from tailoring clothes to motion of Planets - Mathematics applies everywhere. Mathematics as a discipline has its own importance as it prepares a learner to develop problem-solving skill. The present curriculum in Mathematics has six core modules namely Algebra, Commercial Mathematics, Geometry, Mensuration, Trigonmetry and statistics which are distributed over two books namely Book - 1 and Book - 2. Besides these, Laboratory manual is also provided with 30 activities in mathematics for making the learning more effective. The Learner Guide aims to initiate concrete thinking process and enables the learner to explore the content with real life situation.

Objective of the Learner's Guide

1. To facilitate revision of the study materials in a short time
2. To strengthen the learning of the content material
3. To support the learners to enhance their performance in examination
4. To highlight the important concepts and points of information.

Tutor Marked Assignments (TMA)

1. Significance of Tutor Marked Assignments (TMAs) for you

Needless to say that there is great significance of Tutor Marked Assignments (TMAs) in open learning system. In fact, TMAs are an essential and integral part of open learning system. You will get an opportunity to come into contact with your tutor or teacher through TMAs. It provides you an opportunity to know your shortcomings and improve upon the answers. The suggestions/directions of the tutor help you to make the required improvement in the assignments submitted. This would help you prepare for better performance in your examinations.

2. How to prepare a Good Assignments

While preparing assignments focus on the question. The questions usually cover the content of a number of lessons. Give required weightage to content from all the lessons. Write the assignments giving headings and sub-headings. Make sure that all the important information is covered. The assignments should be in conformity with the prescribed format. It should neither be too lengthy nor too small.

3. Responding to the comments of Tutors

Tutors comments will enable you to improve and update your knowledge of the subject. It will help you to correct/rectify your mistakes or lapses. The comments of the Tutors will also help you to prepare yourself for better performance in examinations. It is, therefore, imperative and in your interest to respond positively to the comments of tutors.

Preparing for Examinations

- **Positive side of the Examinations**

  The positive side of Examination is that it provides the examinee (the learner) an opportunity to assess his/her knowledge of the concerned subject and also the level of his competence and capability.

- **Myth about Examinations**

  The myth about the Examinations is that it is the only and sole yardstick to measure, assess and Judge the ability, calibre and competency level of Examinee. The truth or reality is that out of many other techniques, examinations are only one such technique.

- **What to Avoid**

  While preparing for Examinations avoid putting unnecessary stress on your mind to avoid Examination Fear. Do not waste much time in cramming all the details and concentrate on the main points of each lesson or the study material. We have tried to bring these points to you through this Learners' Guide.

- **Revising for Examinations**

  Revising all that you have studied is a must while preparing for Examination. Revision provides an opportunity to recall all that has been studied so far. It also enables you to recollect at least the main points of each lesson or the study material.

- **Tips for preparing for Examinations**

  The time before Examinations is the most crucial for every learner. Some tips to help you better prepare for Examinations are:

  (i) Do revise your lesson/study material

  (ii) Maintain the required level of self confidence

  (iii) Do not allow yourself to suffer from Examination Fear

  (iv) Do reach your Examination Centre well in time

  (v) Keep in mind that you have to complete the answers of all the questions well before the allotted time so that there is enough time for revision of the answer book and ensure that all the questions have been answered.

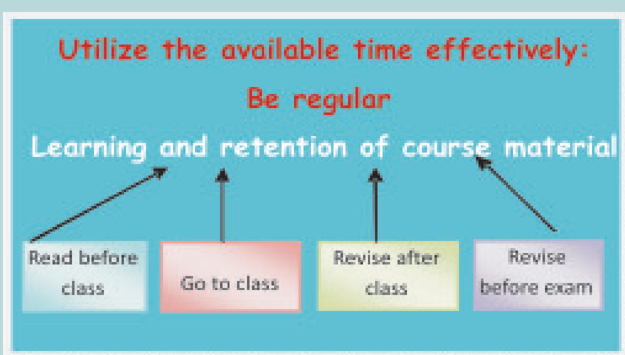
Time Management
before and during examination

Does it sound familiar? Most of students try to postpone the work till last minute and do poor work and also get stressed. Will it help if you plan your time and work systematically?

Ever heard the saying, "Manage your time, or it will manage you"? This is true. On the other hand, you can't really manage time, because it is at no one's command – everyone has 24 hours a day, 168 hours a week. So, **you can only manage yourself around the time.**

Parkinson's Law: Work expands to fill up the available time! You can do lot many tasks if you plan them well.

Benefits of Time Management	
Reduces stress : Preparing over a period of time is less stressful than trying to cram an entire course in few hours before the exam!	**Makes life balanced :** Studying all the time does not mean that you are a 'good' student. You also have other things to do as well as time to relax is important for all students.
Increased output : Working long hours lead to slow speed and tiredness. Utilize your time more effectively. Plan to complete tasks within specific time period.	**Meet goals :** Setting goals is a powerful way of motivating yourself to work. It also helps you reduce postponing and stressed over unfinished work.



Studying for exams!: Tips for better time management

Plan in small blocks

E.g., plan for an hour. You will only be able to really concentrate for a maximum of 45 minutes, so plan a 15-minute break after that.

Plan with exactness

- Indicate exactly what you plan to achieve within that time.
- Example of planning without exactness:
  - English 2-4pm;
  - Biology 5-7pm.
- Example of planning with exactness:
  - Mathematics
  - Paintng Practical – Object Study

Plan with the end in mind

- Start from your goal. Check your exam time table and work backward from there.
- To make to the exam, set specific targets to complete by each week.

Need a weekly planner?

Plan with your strengths in mind

- When are you most productive, or at your mental best – morning, afternoon, or night? Use these times to study your more difficult subjects.
- Use your down-times to do more mechanical

tasks, such as washing, cooking, or shopping (but don't get carried away!).

You probably would not need to divide your time equally between all your subjects. In deciding how much time you want to allocate for each subject, consider the following:

– Amount of study you have done during the term!
– How difficult you perceive the subject to be?
– Weightage of the exam!
– How well you hope to do in it?

Get started with a blank daily planner

Plan with flexibility

- You shouldn't plan a time-table that's so packed, that it leaves you with no cushion time to perform everyday activities (you still need to eat, rest and take bath!) and to deal with unforeseen emergencies.

Reward yourself!

- After you have accomplished each of the tasks you have set out to do, give yourself a break – go for a walk, watch some television, or catch up with your friend.

Managing time for Writing Exam :

Allocate Your Time

- **Look at how marks are allocated**. The number of marks given to a particular question will give you an indication of how much time to spend on it. Look at:

✓ the number of marks per question
✓ how they are distributed
✓ how many questions you have to answer

Ration your time accordingly.

Choose 'easy' or 'difficult' questions.

Deciding the order of questions to answer?

- It's individual preference, some students like to answer short answer questions first and there are others like to answer longest question in the beginning.
- If you want to start with longest, then time it. Do not be tempted to spend extra time.
- Leave your worst question until last. BUT ensure you leave yourself enough time to answer it.
- Devote any extra time to your best questions.
- **Make a note of how much time you should give to each question**. Once you decide on your time outline, stick to it. **Watch the clock**, and once the allocated time has elapsed, stop and move onto the next question.

Are you panicking or tired?

Allow yourself brief rests in the exam. Loosen up physically, stretch (if you can do so without feeling awkward), take several deeper breaths; shut your eyes when you are thinking.

- **If you haven't finished, leave lots of space in the exam booklet**. If you have any extra time at the end (or during the revision period) you can return and answer it more fully.
- Do leave time to check and polish your answers at the very end.
- **Don't leave the exam early**. Use extra time to revise or to think more deeply about one of the harder questions. Make use of all the allocated time - it's worth it.

HOW TO ANSWER QUESTIONS

Strategies for Answering Questions

1. Read the entire question paper.
2. Read the directions carefully.
3. Plan your time accordingly.
4. Jot down anything that comes to your mind while reading the question, so that you do not forget it.
5. Before answering, read the question thoroughly. Number the parts if any and make an outline of the answer so that you do not miss any point.
6. Restate the question as the first line of your answer
7. Do not go into irrelevant details.
8. If you are unsure or get stuck on a question, move on.

Questions which require longer answers, whether in the form of paragraph or essay, focus on direction words. A list of possible words and what they mean is given below:

1. Words asking you to state everything you know about the question

- **Prove :** Demonstrate the truth or existence of something by evidence or logical argumentation. Eg. prove that $\sqrt{2}$ is an irrational number.
- **Solve :** To work out the answer or solution to a mathematical problem. In order to solve a problem. It is essential to know what are given? What is to find out? and what are the essential data require to find out the solution of the problem. Eg. Solve $x^2 - 5x = 6$.
- **Evaluate :** To calculate the numerical value of or examine and judges carefully. Eg. Evaluate $\tan^2 45° - \sin^2 60°$.
- **Explain** : Make clear; interprete, tell 'how' to do. E.g. zero is a rational number.
- **Derive :** Obtain something from the specific source or base a concept on a logical extension of another concept. Eg. derive sum of first *n* terms of an A.P.

3. Words asking for specific characteristics or certain limited facts

- **Compare**- Bring out the points of similarity and the points of differences
- **Contrast**- Bring out the points of difference.
- **Define-** Give the meaning of a word or concept; place it in the class to which it belongs and set it off from other items in the same class.
- **Relate**- Show how things are connected or correlated with the answer
- **Interpret-** Translate; give example of; comment on a subject

3. Words asking for your supported opinion

- **Criticize**- State your opinion of the correctness or merits of an item or issue. Criticism may approve or disapprove
- **Evaluate-** Give the good and bad points; give an opinion regarding the

value of discussing the advantages and disadvantages

- **Justify-** Prove or give reasons for decisions or conclusion.
- **Prove-** Establish that something is true by citing factual evidence or giving clear and logical reasons.

Length of answers

How much to write is most often given in the question. Therefore, reading the directions is absolutely essential.

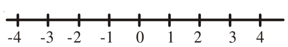
– In some questions especially essay type, the very fact that it has the maximum marks assigned to it, points to a long answer.

– Where it is clearly stated to write a paragraph or two lines, this should be adhered to.

– Word limit if given is also an indication and should be followed.

1

NUMBER SYSTEM

- **Natural Numbers (N):** Counting numbers 1, 2, 3, 4, ...Smallest natural number is 1
- **Whole Numbers (W):** Natural numbers including 0 i.e. 0, 1, 2, 3, 4 .... Smallest whole number is 0
- **Integers (I):** Whole numbers including negatives of natural numbers i.e. ....-3, -2, -1, 0, 1, 2, 3 ....
- **Number Line :** Line on which numbers are represented i.e. 

- **Rational Numbers (Q):** Number p/q is a rational number if p and q are integers and $q \neq 0$.
- **Standard form of a rational number:** p/q is said to be in standard form if q is positive and p and q are co-primes.

Important Result : Every integer is a rational number but every rational number is not an integer. Every fraction is a rational number but vice-versa is not always true

- **Equivalent form of a rational number :** Two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$ are said to be equivalent if $ps = rq$
- **Rational numbers on number line:** Every rational number can be represented on a number line. Coresponding to each rational number, there exists a unique point on the number line but converse is not always true.
- **Comparision of rational numbers :** Reduce the numbers with the same denominator and compare their numerators. On a number line the greater rational number lies to the right of the smaller.
- **Addition of rational numbers:**

  If $\frac{a}{b}$ and $\frac{c}{b}$ are two rational numbers then

  $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$. For $\frac{a}{b}$ and $\frac{c}{d}$,

  $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$, for rational numbers p and q, $p + q = q + p$, for rational number p, $p+0=p=0+p$.
- **Subtraction of rational numbers:** For two rational numbers

  $\frac{a}{b}$ and $\frac{c}{b}$, $\frac{a}{b}-\frac{c}{b}, = \frac{a+(-c)}{b}$, for

  $\frac{a}{b}$ and $\frac{c}{d}$ $=\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$, for p and q,

  $p - q \neq q-p$, for rational number p, $p - 0 = p$
- **Multiplication of rational Numbers:** For two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$, for rational numbers p and q we have $p\times q = q\times p$, For rational number p, $p\times 0=0$, $p\times 1 = p$
- **Division of Rational numbers:** For two rational numbers

  $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$

  For rational numbers p and q, $p\div q \neq q\div p$, for rational number p, $p\div 1 = p$, $p\div(-1) = -p$, $p \div p = 1$, $p \div (-p) = -1$
- **Decimal representation of rational numbers:** Process of expressing a rational number into decimal form is to carry out the process of long division using decimal.

Rational number is either a terminating decimal or a non-terminating repreating decimal.

- **Rational numbers between two rational numbers:** There exists infinitely many rational numbers between two rational numbers.

  A rational number between two rational numbers can be found by calculating the average of given numbers.

- **Irrational numbers:** A decimal expression which is neither terminating nor repeating represents an irrational number.

  Numbers other than rational numbers $\sqrt{2}, \sqrt{3}, \sqrt{5}.0.12345.......$, etc are examples of irrational numbers.

- **Real Numbers:** Rational and Irrational numbers together constitute the system of real.

- **Irrational number between two rational numbers:** If q_1 and q_2 are two rational numbers then an irrational number between them is $\sqrt{q_1 \times q_2}$, Where $q_1 \times q_2$ is not a perfect square. If $q_1 \times q_2$ is a perfect square, then take a number q between q_1 and q_2 such that $q_1 \times q$ or $q \times q_2$ are not perfect squares

  $\Rightarrow \quad \sqrt{q_1 \times q}$ or $\sqrt{q \times q_2}$ is the required irrational number.

- **Irrational number between a rational and irrational number or between two irrational numbers:** Average of both numbers

- **Rounding off numbers:** To round off a number to a given number of decimal places, observe the next digit in the decimal part of the number and proceed as under, if the digit is 5 or more than 5, we add 1 to the precedding digit .

  If the digit is less than 5, ignore it.

CHECK YOUR PROGRESS:

1. The rational number $\frac{-21}{49}$ in lowest terms is :

   (A) $\frac{3}{7}$ (B) $\frac{-3}{7}$ (C) $\frac{-7}{3}$ (D) -3

2. $3.\overline{4}$ can be written in the form $\frac{p}{q}$ as:

   (A) $\frac{13}{4}$ (B) $\frac{4}{3}$ (C) $\frac{9}{31}$ (D) $\frac{31}{9}$

3. Number of rational numbers which lie between 2 and 7 is :

   (A) 5 (B) 6 (C) 7 (D) Infinitely many

4. An irrational number lying between $\sqrt{3}$ and 3 is:

   (A) $\sqrt{4}$ (B) $\sqrt{10}$ (C) $\sqrt{5}$ (D) $2\sqrt{3}$

5. Which of the following is not a rational number?

   (A) $\frac{\sqrt{3}}{2}$ (B) 3 (C) $\frac{5}{2}$ (D) $\frac{-3}{5}$

6. Find two rational numbers between 1.23 and 1.24.
7. Simplify: $(\sqrt{32}\times\sqrt{50})\times\sqrt{72}\div 36\sqrt{8}$.
8. Find three irrational numbers between 3 and 4.
9. Represent the following rational numbers on number line

   (A) $\frac{7}{2}$ (B) $\frac{-18}{5}$
10. Represent the following irrational numbers on number line

   (A) $\sqrt{3}$ (B) $\sqrt{7}$

STRETCH YOURSELF :

1. By fnding the decimal representation of $\frac{22}{7}$, comment, is it rational or irrational? Find its apporoximate value up to three places of decimals.
2. Comment, 0 is a rational number or not. Justify your answer.

ANSWERS

CHECK YOUR PROGRESS:

1. B 2. D 3. D 4. C

5. A 6. 1.2325, 1.235 7. $\frac{10}{3}$

8. $2\sqrt{3}$, $\frac{3+2\sqrt{3}}{2}$, $\sqrt{3}+2$

STRETCH YOURSELF:

1. $\frac{22}{7}=3.\overline{142857}$, so it is a rational number, approximate value is 3.143.
2. Yes, Zero is a rational number because 0 can be written as $\frac{0}{\text{any non zero integer}}$

2

EXPONENTS AND RADICALS

- **Expoential Notion:** The notation for writing the product of a number by itself several times e.g. $a \times a \times a \times a = a^4$
- **Base and Exponent:** $(a)^n = a \times a \times a....$ n times, a = base, n = exponent
- **Reading of an exponent:** $5 \times 5 \times 5 \times .....$ 20 times = 5^{20} is read as '20th powers of 5' or 5 raised to the power 20.
- **Prime factorisation:** Any natural number other then1, can be expressed as a product of power of prime numbers.
- **Laws of exponents:** $a^m \times a^n = a^{m+n}$ a $a \neq 0$

  $a^m \div n^a = a^{m-n}$ (if $m > n$),

  $a^m \div a^n = \frac{1}{a^{n-m}}$ (if $m < n$)

  $(a^m)^n = a^{mn}$, $a \neq 0$

  $a^0 = 1$, $a \neq 0$

  $(a \times b)^m = a^m \times b^m$, $a \neq 0, b \neq 0$

  $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $a \neq 0, b \neq 0$
- **Negative integers as exponents :** a is non-zero rational number and m is any positive integer, then the reciprocal of a^{-m} is written as a^m or $\frac{1}{a^m}$ and read as 'a raised to the power (–m) or $\frac{1}{a^m} = a^{-m}$
- **Radicals or surds:** $\sqrt[n]{x}$ is a surd if and only if it is an irrational number and it is a root of the positive rational number. $\sqrt{\ }$ is called a radical sign. The index n is called the order of the surd and x is called the radicand.
- **Pure and mixed surd :** A surd with rational factor 1 only other factor being irrational is called pure surd e.g. $\sqrt[5]{16}, \sqrt[3]{50}$.

  A surd having rational factor other then 1 alongwith the irrational factor is called a mixed surd e.g. $5\sqrt[2]{3}, 4\sqrt[3]{7}$
- **Laws of surds:** If x, y are the positive rational numbers and m, n and p are positive integers then $\left(\sqrt[n]{x}\right)^n = x$

  $\sqrt[n]{x}.\sqrt[n]{y} = \sqrt[n]{xy}$ or $x^{\frac{1}{n}}.y^{\frac{1}{n}} = (xy)^{\frac{1}{n}}$

  $\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$ or $\frac{x^{1/n}}{y^{1/n}} = \left(\frac{x}{y}\right)^{\frac{1}{n}}$

  $\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x} = \sqrt[n]{\sqrt[m]{x}}$ or

  $\left(x^{\frac{1}{n}}\right)^{\frac{1}{m}} = x^{\frac{1}{mn}} = \left(x^{\frac{1}{m}}\right)^{\frac{1}{n}}$

  $\sqrt[n]{x^m} = x^{\frac{m}{n}}$ or $(x^m)^{\frac{1}{n}} = x^{\frac{m}{n}}$

  $\sqrt[n]{x^p} = \sqrt[mn]{x^{mp}}$ or $x^{\frac{pm}{mn}} = (x^{pm})^{\frac{1}{mn}}$
- **Similar or like surds:** Two sruds are said to be similar if they have same irrational factor e.g $3\sqrt{5}$ and $7\sqrt{5}$ are like or similar surds.
- **Simplest or lowest form of a surd :** A surd is said to be in simplest form, if it has Smallest possible index of the sign, no fraction under radical sign, no factor of the form a^n where a is a positive integer under the radical sign of index n.
- **Properties of surds:** Similar surds can be subtracted and added

  Order of surds can be changed by multiplying index of the surds and index of the radinard by the same positive number surds of the same order can be multiplided and divided

- **Comparision fo Surds:** Change the given surds to surds of the same order, then compare their radicands alongwith co-efficients.
- **Rationalising factor of a surd:** If the product of the two surds is rational, each is called the rationalising factor of the other $x+\sqrt{y}$ is called the rationalising factor of $x-\sqrt{y}$ and vice-versa

CHECK YOUR PROGRESS:

1. $\left(-\frac{2}{3}\right)^3 \times \left(-\frac{2}{3}\right)^5$ equals to:

   (A) $\left(-\frac{2}{3}\right)^{15}$ (B) $\left(\frac{2}{3}\right)^{-15}$ (C) $\left(-\frac{2}{3}\right)^{8}$ (D) $\left(-\frac{2}{3}\right)^{2}$

2. The order of the surd $3\sqrt[5]{47}$ is:

   (A) 5 (B) 3 (C) 47 (D) $\frac{1}{5}$

3. The rationalising factor of $\sqrt[3]{25}$ is :

   (A) 5 (B) $\sqrt{5}$ (C) $\sqrt[3]{5}$ (D) $\sqrt[3]{25}$

4. $\sqrt{8}$ is a:

   (A) Pure Surd (B) Mixed Sured (C) Not a Surd (D) Rational Number

5. $\left(-\frac{3}{4}\right)^0$ is equal to :

   (A) –1 (B) 1 (C) $\frac{-3}{4}$ (D) $\frac{-4}{3}$

6. Express the following as a product of prime factors in exponential form:

   (A) 194400 (B) 864360

7. Express the following as a mixed surd in simplest form

   (A) $\sqrt[4]{1215}$ (B) $\sqrt[3]{1024}$

8. Express the following as a pure surd:

   (A) $5\sqrt{2}$ (B) $4\sqrt[3]{5}$ (C) $2\sqrt[5]{2}$

9. Simplify each of the following:

   (i) $3\sqrt{80}-\frac{3}{2}\sqrt{\frac{1}{5}}+3\sqrt{120}$ (ii) $2\sqrt{50}\times\sqrt{32}\times2\sqrt{72}$ (iii) $\frac{15\sqrt[3]{13}}{6\sqrt[6]{5}}$

10. (i) Arrange in ascending order: $\sqrt[3]{2}, \sqrt{3}$ and $\sqrt[6]{5}$

(ii) Arrange in descending order : $\sqrt[3]{2}, \sqrt[4]{3}, \sqrt[3]{4}$

11. Simplify the following by rationalising the denominator

(i) $\frac{28}{\sqrt{7}+\sqrt{3}}$ (ii) $\frac{\sqrt{7}-2}{\sqrt{7}+2}$ (iii) $\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$

STRETCH YOURSELF :

1. Find the value of x, if $\left(\frac{5}{7}\right)^{5+x} \times \left(\frac{25}{49}\right)^{x} = \left(\frac{7}{5}\right)^{2}$.

2. Simplify: $\left(\frac{-5}{6}\right)^{2} \div \left(\frac{-3}{5}\right)^{2}$.

3. if $x = 7+4\sqrt{3}$, Find the value of $x+\frac{1}{x}$.

4. If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$, Find the values of a and b.

ANSWERS

CHECK YOUR PROGRESS:

1. C 2. A 3. C

4. B 5. B

6. (i) $2^5\, 3^5\, 5^2$ (ii) $2^5\, 3^2\, 5^1\, 7^4$

7. (i) $3\sqrt[4]{15}$ (ii) $8\sqrt[3]{2}$

8. (i) $\sqrt{50}$ (ii) $\sqrt[3]{320}$

(iii) $\sqrt[5]{64}$

9. (i) $\frac{88}{5}\sqrt{5}$ (ii) $960\sqrt{2}$

(iii) $\frac{1}{2}\sqrt[6]{845}$

10. (i) $\sqrt[3]{2}$, $\sqrt[6]{5}$, $\sqrt{3}$

(ii) $\sqrt[3]{4}$, $\sqrt[4]{3}$, $\sqrt[3]{2}$

11. (i) $7\left(\sqrt{7}-\sqrt{3}\right)$ (ii) $\frac{11-4\sqrt{7}}{3}$

(iii) $\frac{2\sqrt{3}+3\sqrt{2}+\sqrt{30}}{12}$

STRETCH YOURSELF

1. x = 7

2. $\frac{625}{324}$

3. 14

4. a = 11, b = –6

3

ALGEBRAIC EXPRESSIONS AND POLYNOMIALS

- **Constant:** Quantity which has a fixed numerical value e.g. 0, 1, 2 ....
- **Variable:** Quantity which can take different numerical values .A variable is represented by a letter of the English alphabet such as a, b, c, x, y, z etc.
- **Algebraic expressions:** A combination of constants and variables, connected by any or all of the four fundamental operations (+, -, ×, ÷).
- **Term:** Each part of the expression alongwith its sign
- **Monomial:** An algebraic expression containing one term eg $6a^2$, $3x^2y^2$ etc.
- **Binomial:** An algebraic expression containing two terms e.g. $a^2 + b^2$, $7xy + y^2$ etc.
- **Trinomial:** An algebraic expression containing three terms e.g. $x^2 + y^2 + z^2$, $x^2 + 2xy + y^2$ etc.
- **Polynomial:** An algebraic expression in which variable(s) does (do) not occur in the denominator, exponents of variables are whole numbers and numerical cofficients of various terms are real numbers e.g. $x^3 - 2y^2 + y - \sqrt{7}$ is a polynomial while $x^3 - \frac{1}{x}$ is not a polynomial.
- **Factor:** When two or more numbers or variables are multiplied, then each one of them and their product is called a factor of the product. A constant factor is a numerical factor while a variable is known as a literal factor.
- **Coefficient:** In a term any one of the facotrs with the sign of the term is the coefficient of the product of the other factors e.g. in $-3xy$, cofficient of x is $-3y$.
- **Constant Term:** Term which has no literal factor e.g. in $2x + 9y + 7$ the constant term is 7.
- **Like and Unlike Terms:** Terms having same literal factors are called like or similar terms and terms having different literal factors are called unlike terms.
- **Degree of a polynomial :** Sum of the exponents of the variables in a term is called degree of the term.

  Degree of a polynomial is the same as the degree of its term or terms having the highest degree and non-zero coefficient.
- **Quadratic polynomial :** A polynomial of degree2 e.g. $x^2 - 3x + 2$.
- **Zero degree polynomial:** Degree of a non-zero constant polynomial is taken as zero

  Zero polynomial : When all the coefficients of variables in the terms of a polynomial are zeros, the polynomial is called a zero polynomial and the degree of zero polynomial is not defined.
- **Zeros of a polynomial:** Value(s) of the variable for which the value of a polynomial in one variable is zero.
- **Addition and subtraction of polynomials:** The sum of two (or more) like terms is a like term whose numerical coefficient is the sum of the numerical coefficients of the like terms

  The difference of two like terms is a like term whose numerical coefficient is the difference of the numerical coefficients of the like terms

  To add polynomials, add their like terms together e.g. $2x + 3x = 5x$, $3x^2y + 8x^2y = 11x^2y$

  To subtract a polynomial from another polynomial subtract a term from a like term e.g $9x^2y^2 - 5x^2y^2 = 4x^2y^2$, $5y - 2y = 3y$.
- **Multiplication of the polynomials:** To multiply a monomial by a monomial, use laws of

exponents and the rules of the signs e.g. $3a \times a^2b^2 = 3a^3b^2$

To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial.

To multiply a polynomial by another polynomial multiply each term of the one polynomial by each term of the other polynomial and simplify the the result by combining like terms.

- **Division of polynomials:** To divide a monomial by another monomial, find the quotient of numerical cofficients and variables separately using laws of exponents and then multiply these quotients.

  To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

  Process of division of a ploynomial by another polynomial is done on similar lines as in arithmatic after arranging the terms of both polynomials In decreasing powers of the variable common to both of them.

  If remainder is zero the divisor is a factor of dividend.

  Dividend = Divisor x quotient + Remainder.

CHECK YOUR PROGRESS:

1. The degree of a non zero constant is :

   (A) 0 (B) 1 (C) 2 (D) 3

2. The coefficient of x^5 in $7x^5y^3$ is :

   (A) 7 (B) 4^3 (C) $7y^3$ (D) 5

3. The degree of the polynomial $5x^6y^4 + x^2y + xy^2 - 3xy + 4$ is:

   (A) 2 (B) 3 (C) 6 (D)10

4. Which of the following is a polynomial?

   (A) $x^2 - 5\sqrt{x} + 2$ (B) $\sqrt{x} + \frac{1}{\sqrt{x}}$ (C) $\frac{5}{x^2 - 3x + 1}$ (D) None of these

5. A zero of the polynomial $x^2 - 2x - 15$ is:

   (A) –5 (B) –3 (C) 0 (D) 3

6. Which of the following pairs of terms is a pair of like terms?

   (A) 2a, 2b (B) $2xy^3, 2x^3y$ (C) $3x^2y, \frac{1}{\sqrt{2}}yx^2$ (D) 8, 16a

7. Add $\frac{2}{3}x^2 + x + 1$ and $\frac{3}{7}x^2 + \frac{1}{4}x + 2$.

8. Subtract $7x^3 - 3x^2 + 2$ from $x^2 - 5x + 2$.

9. Find the product of $(2x + 3)$ and $(x^2 - 3x + 4)$.

10. Find the quotient and remainder when $6x^2 - 5x + 1$ is divided by $2x - 1$.

11. Evaluate $3xy - x^3 - y^3 + z^3$ at $x = 2, y = 1, z = -3$.

STRETCH YOURSELF :

1. What should be added to $x^2 + xy + y^2$ to get $2x^2 + 3xy$.
2. What should be subtracted from $-13x + 5y - 8$ to get $11x - 16y + 7$?
3. Subtract the product of $(x^2 - xy + y^2)$ and $(x + y)$ from the product of $(x^2 + xy + y^2)$ and $(x - y)$. What is the coefficient of x^3 in the product?
4. Subtract $3x - y - xy$ from the sum of $3x - y + 2xy$ and $-y - xy$. What is the coefficient of x in result?

ANSWERS

CHECK YOUR PROGRESS:

1.	A	2. C	3. D
4.	D	5. B	6. C

7. $\frac{23}{21}x^2 + \frac{5}{4}x + 6$ 8. $-7x^3 + 4x^2 - 5x$

9. $2x^3 - 3x^2 - x + 12$

10. Quotient $= 3x - 1$, Remainder $= 0$

11. -54

STRETCH YOURSELF:

1. $x^2 + 2xy - y^2$ 2. $-24x + 21y - 15$

3. $-2y^3, 0$ 4. $2xy - y, 2y$

4

SPECIAL PRODUCTS AND FACTORIZATION

- **Special Products:** Products like 108×108, 97×97, 104×96 can easily be calculated with the help of $(a + b)^2$, $(a - b)^2$, $(a + b)(a - b)$ respectively. Such products are called special products.

Special Product Formula :

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

$(a + b)^2 - (a - b)^2 = 4ab$

$(a + b)(a - b) = a^2 - b^2$

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$(x - a)(x - b) = x^2 - (a + b)x + ab$

$(a - b)^3 = a^3 - 3ab(a - b) - b^3$

$(a + b)^3 = a^3 + 3ab(a + b) + b^3$

$(a + b)(a^2 - ab + b^2) = a^3 + b^3$

$(a - b)(a^2 + ab + b^2) = a^3 - b^3$

- **Factorization of polynomials:** Factorization of polynomials is a process of writing the polynomial as a product of two (or more) polynomials. Each polynomial in the product is called a factor of the given polynomial.
- **Method of factorization:** Factorization by distributive property.

  Factorization involving the difference of two squares.

  Factorizaion of a perfect square polynomial.

  Factorization of a polynomial reducible to the difference of two squares.

  Factorization of perfect cube polynomials.

  Factorization of polynomials involving sum or difference of two cubes.

  Factorizing trinomials by splitting the middle term.
- **HCF of polynomials:** HCF of two or more given polynomials is the product of the polynomials of highest degree and greatest numerical coefficient each of which is a factor of each of the given polynomials.
- **LCM of polynomials:** LCM of two or more polynomials is the product of the polynomials of the lowest degree and the smallest numerical coefficient which are multiples of the corresponding elements of each of the given polynomials.
- **Rational Expression:** An algebraic expression which can be expressed in the form $\frac{p}{q}$ where p is any polynomial and q is non-zero polynomial. A rational expression need not to be a polynomial. Every polynomial is a rational expression also.
- **Operations on rational expressions:** Four fundamental operations $(+, -, \times, \div)$ on rational expressions are performed in exactly the same way as in the case of rational numbers.

  Result of multiplication of rational expressions must be in the lowest terms or in lowest form.

  Sum, difference, product and quotient of two rational expressions are also rational expressions.
- **Reciprocal expression:** $\frac{S}{R}$ is the reciprocal expression of $\frac{R}{S}$.

  We use reciprocal expression in division of two rational expressions as $\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \times \frac{S}{R}$.
- **Conversion of a rational expression into lowest terms:** Cancel the common factor if any from the numerator and denominator of the rational expression.

CHECK YOUR PROGRESS:

1. Which of the following is not a rational expression?

   (A) $\sqrt{21}$ (B) $x+\frac{1}{x}$ (C) $8\sqrt{x}+6\sqrt{y}$ (D) $\frac{x+\sqrt{2}}{x-\sqrt{2}}$

2. $\left(a^2+b^2\right)^2+\left(a^2-b^2\right)^2$ equals to :

   (A) $2\left(a^2+b^2\right)$ (B) $4\left(a^2+b^2\right)$ (C) $4\left(a^4+b^4\right)$ (D) $2\left(a^4+b^4\right)$

3. If $m-\frac{1}{m}=-\sqrt{3}$, then $m^3-\frac{1}{m^3}$ equals to :

   (A) $-6\sqrt{3}$ (B) $-3\sqrt{3}$ (C) 0 (D) $6\sqrt{3}$

4. $\frac{327\times327-323\times323}{327+323}$ equals to:

   (A) 650 (B) 327 (C) 323 (D) 4

5. $8m^3 - n^3$ equals to:

   (A) $(2m-n)\left(4m^2-2mn+n^2\right)$

   (B) $(2m-n)\left(4m^2+2mn+n^2\right)$

   (C) $(2m-n)\left(4m^2+4mn+n^2\right)$

   (D) $(2m+n)\left(4m^2+2mn+n^2\right)$

6. Find the sum of $\frac{x+2}{x-2}$ and $\frac{x-2}{x+2}$

7. Find the LCM of x^2-1 and x^2-x-2.

8. Find the HCF of $36x^5y^2$ and $90x^3y^4$.

9. Factorise (i) x^4-81y^4 (ii) $5x^2-8x-4$.

10. Simplify the following:

    $$\frac{6x^2+17x+12}{10x^2+17x+3}\div\frac{6x^2-7x-20}{10x^2-23x-5}.$$

STRETCH YOURSELF :

1. If $a^4 + \frac{1}{a^4} = 34$, find $a^3 - \frac{1}{a^3}$.
2. Find the sum of $\frac{x+1}{x-1}$ and its reciprocal.
3. Without actual multiplication evaluate $103 \times 103 \times 103$.
4. Find the value of $x^3 - y^3$ when $x - y = 5$ and $xy = 66$.

ANSWERS

CHECK YOUR PROGRESS:

1. C   2. D   3. A
4. D   5. B
6. $\frac{2x^2+8}{x^2-4}$ 7. $(x^2-1)(x-2)$
8. $18x^3y^2$
9. (i) $(x^2+9y^2)\ (x+3y)\ (x-3y)$

   (ii) $(x-2)\ (5x+2)$
10. 1

STRETCH YOURSELF :

1. 14   2. $\frac{2(x^2+1)}{(x^2-1)}$
3. 1092727   4. 1115

5

LINEAR EQUATIONS

- **Linear polynomial:** A polynomial having degree1.
- **Equation:** Two expressions separated by sign of equality.
- **Linear Equation:** Equation involving only linear polynomials.

  An equaltion in which the highest power of the variable is 1.

  General form of a linear equation in one variable is $ax + b = 0$, $a \neq 0$, and a, b are real numbers.
- **Left Hand Side (LHS) :** Expression to the left of the equality sign.
- **Right Hand side (RHS) :** Expression to the right of the equality sign.
- **Solution of linear equation in one variable:** The value of the variable for which LHS of the given equation becomes equal to RHS.
- **Rules for solving an equation:** Same number can be added to both sides of the equation.

  Same number can be subtracted from both sides of the equation.

  Both sides of the equation can be multiplied by the same non-zero number.

  Both the sides of the equation can be divided by the same non-zero number.
- **Transposition:** Process by which any term of the equation can be taken from one side of the equality to the other side by changing its sign.
- **Formation of a linear equation in one variable:** Represent the unknown by an alphabet say x, y, z, m, n, p etc. and translate the given statement into an equation.
- **Linear equation in two variables :** $ax + by + c = 0$ is a linear equation in two variables x and y. Linear equation in two variables have infinitely many solutions.

  In $ax + by + c = 0$ for each value of y, we get a unique value of x.

  $$\Rightarrow ax = -by - c \Rightarrow x = \frac{-by}{a} - \frac{c}{a}$$

  A linear equation $ax + c = 0$, $a \neq 0$ can be considered as linear equation in two variables by expressing it as $ax + 0.y + c = 0$.
- **Graph of a linear equation in two variables:** Find at least two points in the plane whose coordinates are solutions of the equation. Plot them on coordinate plane and join them using scale.

  Graph of a linear equation in two variables is always a straight line.
- **System of linear equations :** A pair of linear equations in two variables is said to form a system of linear equations written as

  $a_1x+b_1y+c_1=0$ (a_1, b_1 both $\neq 0$)

  $a_2x + b_2y + c_2 = 0$ (a_2, b_2 both $\neq 0$) where a_1, a_2, b_1, b_2, c_1, c_2 are real numbers.

  System of linear equations can be solved by graphical or any algebraic method.
- **Graphical method for solution of system of linear equations:** Draw the graph of both equations on same graph paper.

  If the graph is intersecting lines then the point of intersection gives unique solution of system.

  If two lines coincide, system has infinitely many solutions.

  If graph is parallel lines, the system has no solution.
- **Algebraic method for Solution of system of linear equations:** Substitution Method: Find the value of one variable in terms of other variable from one equation and substitute it in second equation, second equation will be reduced to linear equation in one variable.

  Elimination Method: Multiply both equations by suitable non-zero constants to make the coefficients of one of the variables numerically equal. Now add or subtract one equation from another to eliminate one variable, we get an equation in one variable.
- **Word Problems based on linear equations:** Translate the given information (data) into linear equations(s) and solve them.

CHECK YOUR PROGRESS:

1. The degree of a linear equation is:
   (A) 1 (B) 2 (C) 3 (D) 0
2. Which of the following numbers is the solution of $x + 3 = 9$?
   (A) 3 (B) 6 (C) 9 (D) 12
3. Which of the following ordered pairs is a point on the straight line represented by $4x - 3y + 1 = 0$?
   (A) (2, 1) (B) (5, 3) (C) (3, 2) (D) (5, 7)
4. If the point (K, 4) lies on the straight line represented by $3x + y = 10$, then the value of K is :
   (A) 1 (B) 2 (C) 3 (D) 4
5. A system of linear equations in two variables has unique solution if the graph is :
   (A) Intersecting lines (B) Coincident (the same) lines
   (C) Parallel lines (D) None of these
6. Solve the following system of linear equations graphically:
   $x - 2y = 7,\ 3x + y = 35.$
7. Solve the following system of linear equations by substitution method:
   $2x + 3y = 13,\ 5x - 7y = -11.$
8. Solve the following pair of equations by elimination method:
   $3x + 2y = 11,\ 2x + 3y = 4$
9. If the numerator of a fraction is decreased by one, it becomes $\frac{2}{3}$ but, if the denominator is increased by 5, the fraction becomes $\frac{1}{2}$. Find the fraction.
10. The perimeter of a rectangle is 20cm. If length exceeds breadth by 4 cm, Find the area of the rectangle.

STRETCH YOURSELF :

1. Draw the graph of $4x + 5y = 20$. Hence show that the point (2, 3) does not lie on the line represented by $4x + 5y = 20$.
2. Solve for p and q:

   $$4p + \frac{6}{q} = 15,$$

   $$6p - \frac{8}{q} = 14.$$
3. Draw the graph of the follwoing pair of equations:
   $2x - y = -8,\ 8x + 3y = 24$
   Determine the vertices of the triangle formed by the lines represented by these equations and x -axis. Shade the triangular region so formed.

ANSWERS

CHECK YOUR PROGRESS:

1. A 2. B 3. D 4. B
5. A 6. $x = 11, y = 2$
7. $x = 2, y = 3$ 8. $x = 5, y = -2$
9. $\frac{7}{9}$ 10. $21cm^2$

STRETCH YOURSELF :

2. $p = 3, q = 2.$
3. (0, 8) (-4, 0) and (3, 0)

6

QUADRATIC EQUATIONS

- **Quadratic polynomial:** A polynomial of degree2
- **Quadratic equation:** An equation having degree 2.
- **General form of a quadratic equation:** $ax^2 + bx + c = 0$, $a \neq 0$ where a, b, c are real numbers and x is a variable.
- **Roots of a quadratic equation:** Values of variable which satisfy a quadratic equation. α is a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

  A quadric equation has two roots.

  Zeros of a quadratic polynomial and the roots of the corresponding quadratic equation are the same.
- **Methods for solution of quadratic equation:**

  (i) Factor method

  (ii) Using the quadratic formula
- **Factor method of solving $ax^2 + bx + c = 0$, $a \neq 0$:** Factorise $ax^2 + bx + c$, $a \neq 0$ into a product of two linear factors. Equate each factor to zero and get the values of the variable. These values are the required roots of the given quadratic equation.
- **Quadratic formula :** The roots of the equation $ax^2 + bx + c = 0$ are

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}.$$

- **Discriminant :** The expression $b^2 - 4ac$ is called discriminant of the equation $ax^2 + bx + c = 0$ and denoted by D.
- **Nature of Roots :** A quadratic equation

  $ax^2 + bx + c = 0$ $(a \neq 0)$ has

  (i) two distinct real roots if $D = b^2 - 4ac > 0$

  (ii) two equal (or coincident) and real roots if $D = b^2 - 4ac = 0$

  (iii) no real root if $D = b^2 - 4ac < 0$.
- **Word Problems or daily life problems:** To solve a word problem using quadratic equations convert the given problem in the form of a quadratic equation and then solve the equation by using factor method or quadratic formula.

CHECK YOUR PROGRESS:

1. Which of the following is not a quadratic equation?

   (A) (x - 1) (x +3) = 6 (B) $x + \frac{1}{x} = 7$

   (C) $3x^2 - 5x + 2 = 0$ (D) $x^2 + 2\sqrt{x} + 3 = 0$

2. If the quadratic equation $3x^2 + mx + 2 = 0$ has real and equal roots, then the value of m is :

   (A) $-\sqrt{6}$ (B) $\sqrt{6}$ (C) $\frac{\sqrt{6}}{2}$ (D) $\pm 2\sqrt{6}$

3. The discriminant of the quadratic equation $5x^2 - 6x - 2 = 0$ is :

   (A) 56 (B) 66 (C) 76 (D) 86

4. If one root of the quadratic equation $x^2 - \alpha x - 5 = 0$ is 5 then the other root is :
   (A) -1 (B) 1 (C) $-\alpha$ (D) α
5. Roots of the quadratic equation $x^2 - 14x + 45 = 0$ are:
   (A) real and equal (B) real and distinct
   (C) not real (D) none of these
6. Solve the following equations by factor method:
   (i) $x^2 + 3x = 18$ (B) $2x^2 + 5x - 3 = 0$
7. Solve the follwoing quadratic equations using quadratic formula:
   (i) $3x^2 - 4x - 7 = 0$ (ii) $6x^2 - 19x + 15 = 0$
8. The sum of the ages (in years) of a father and his son is 60 and the product of their ages is 576. Find their ages.
9. Find two consecutive odd positive integers if the sum of their squares is 290.
10. The product of the digits of a two digit number is 12. When 9 is added to the number, the digits interchange their places. Find the number.

STRETCH YOUSELF

1. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equaion $P(x^2 + x) + k = 0$ has equal roots, find the value of K.
2. Find the value of K for which the quadratic equation $x^2 - 4x + K = 0$ has two real and distinct roots.
3. Solve the equation:

$$\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15},\ x \neq 0, -1.$$

4. If x = 2 and x = 3 are the roots of the equation $3x^2 - 2kx + 2m = 0$, find the values of k and m.
5. Find the value of k for which the quadratic equation $x^2 - 2x(1+3k) + 7(3+2k) = 0$ has real and eqaul roots.

ANSWERS

CHECK YOUR PROGRESS :

1. D 2. D 3. C 4. A
5. B 6. (i) 3, –6 (ii) $\frac{1}{2}, -3$
7. (i) -1, $\frac{7}{3}$ (ii) $\frac{3}{2}, \frac{5}{3}$
8. Father's age = 48 years, son's is age = 12 years.
9. 11, 13 10. 34

STRETCH YOURSELF:

1. $\frac{7}{4}$ 2. K < 4 3. $\left(\frac{-5}{2}, \frac{3}{2}\right)$
4. $k = \frac{15}{2}$, m = 9 5. k = 2 or $k = \frac{-10}{9}$

7

ARITHMETIC PROGRESSION

- **Sequence (Progression):** A group of numbers forming a pattern
- **Arithmetic Progression** (A.P.): A progression in which each term, except the first, is obtained by adding a constant to the previous term. Its terms are denoted by $t_1, t_2, t_3, ... t_n$, or $a_1, a_2, a_3, ...a_n$.

  A squence is called an arithmetic progression, if there exists a constant d such that $a_2 - a_1 = d$, $a_3 - a_2 = d$, $a_4 - a_3 = d$, ... $a_{n+1} - a_n = d$ and so on. d is called the common difference.
- **Formation of A.P. or General form of A.P.:** If 'a' is the first term and 'd' is the common difference of an A.P., then A.P. is a, a + d, a + 2d, a + 3d, a + 4d, ...
- **'n'th term of A.P.:** The nth term of the A.P. a, a + d, a + 2d, ... is given by $t_n = a + (n - 1) d$. Sometimes nth term is also denoted by a_n.
- **Sum of first n terms of an A.P. :** The sum of first n terms of an A.P. is $S_n = \frac{n}{2}(a + l)$, where l (last term) = a + (n – 1) d, a = first term, d = common difference , n = no. of terms

  $\therefore\ s_n = \frac{n}{2}[2a + (n - 1)d]$
- **n^{th} term in terms of s_n:** If s_n is the sum of the first n terms of an A.P., then the nth term is given by $t_n = s_n - s_{n-1}$.
- **Various terms of an A.P.:** 3 consecutive terms are a – d, a, a + d and common difference is d.

  4 consecutive terms are a – 3d, a – d, a + d, a + 3d and common difference is 2 d .

CHECK YOUR PROGRESS:

1. Which of the following progression is an A.P.?

   (A) 1, 4, 9, 16 ..... (B) 1, 3, 9, 27 (C) -2, 0, 2, 4, 6, ... (D) 1, 2, 4, 8, ...

2. The common difference of the A.P. 3, 1, -1, -3, ... is

   (A) -2 (B) 2 (C) –3 (D) 3

3. How many two digit numbers are divisible by 3?

   (A) 31 (B) 30 (C) 29 (D) 11

4. If the first term and common difference of an A.P are 2 and 4 respectively, then the sum of its first 40 terms is :

   (A) 3200 (B) 2800 (C) 1600 (D) 200

5. The sum of the first 10 terms of the A.P. 3, 4, 5, 6, ... is

   (A) 65 (B) 75 (C) 85 (D) 110

6. Find the sum of the A.P, 7 + 12 + 17 + 22 + ... +1002.

7. Find the middle term of the A.P. -11, -7, -3, ..., 53.
8. Which term of the A.P. 9, 14, 19, ... is 124?
9. The 7^{th} and 13^{th} terms of an A.P are 32 and 62 respectively. Find the A.P.
10. Find the 8^{th} term from the end of the A.P. 7, 10, 13, ..., 184.
11. Find the sum of First 25 terms of an A.P. whose nth term is given by $a_n = 2 - 3n$.
12. If 2x, x + 10, 3x + 2 are in A.P. , find the value of x.
13. Which term of the A.P. 3, 15, 27, 39, ... will be 120 more than its 21st term?
14. The sum of 4^{th} and 8^{th} terms of an A.P is 24 and the sum of 6^{th} and 10^{th} terms is 44. Find the A.P.
15. How many terms of the A.P. -10, -7, -4, -1, ... are needed to get the sum 104?

STRETCH YOURSELF:

1. The sum of first n terms of an A.P. is given by $s_n = 3n^2 + 5n$. Find the common difference and 1^{st} term of the A.P.
2. If the 9th term of an A.P is 449 and 449^{th} term is 9, then which term of the A.P. is zero?
3. Which term of the A.P 114, 109, 104, ... is the first negative term?
4. If 7 times the 7^{th} term of an A.P is equal to 11 times the 11^{th} term. show that the 18^{th} term of the A.P. is zero.
5. If p^{th}, q^{th} and r^{th} terms of an A.P. are a, b, c respectively then show that $a(q-r) + b(r-p) + c(p-q) = 0$.

ANSWERS

CHECK YOUR PROGRESS:

1. C	2. A	3. B	4. A
5. B	6. 100900	7. 21	8. 25
9. 2, 7, 12, 17....		10. 163	
11. –925	12. 6	13. 31st	
14. –13, –8, –3.		15. 13	

STRETCH YOURSELF :

1. 6, 8
2. 558^{th}
3. 24^{th}

8

PERCENTAGE AND ITS APPLICATIONS

- **Percentage:** Percent means per every hundred and denoted by the symbol '%' A fraction with denominator 100 is called a 'Percent'.
- **Percent as a fraction:** Drop the % sign and multiply the given number by $1/100$ and simplify it.
- **Percent as a decimal:** Drop the % sign and insert or move the decimal point two places to the left.
- **Fraction as a percent :** Multipy the fraction by 100, simplify it and mark '%' sign.
- **Decimal as a percent:** Shift the decimal point two places to the right and mark '%' sign.
- **Cost Price (c.p.):** Amount paid to buy an article.
- **Selling Pirce (s.p.):** Amount at which an article is sold.
- **Profit or Gain:** When s.p > c.p., the seller makes a profit or gain.

  Gain = s.p. – c.p.
- **Loss:** When c.p. > s.p. , the seller incurs a loss.

  Loss = c.p. – s.p.

  Gain and loss are always calculated on the c.p.
- **Gain %:** Gain on Rs. 100, Gain % = $\frac{\text{Gain}\times 100}{\text{c.p}}$, Overhead expenses are also included in the c.p.
- **Loss %:** Loss on Rs. 100, Loss % = $\frac{\text{Loss}\times 100}{\text{c.p}}$
- **Relation between s.p and c.p:** In case of Gain:

$$\text{c.p.} = \frac{100}{100+\%\text{gain}}\times \text{s.p.}$$

$$\text{s.p.} = \frac{100+\%\text{gain}}{100}\times \text{c.p.}$$

In case of loss: $\text{c.p} = \frac{100}{100-\%\text{loss}}\times \text{s.p}$

$$\text{s.p.} = \frac{100-\%\text{loss}}{100}\times \text{c.p}$$

- **Principal (P):** Money borrowed
- **Interest (I) :** Extra/Addtional money paid by the borrower. S.I. = $\frac{p\times r\times t}{100}$

  $p = \frac{\text{S.I.}\times 100}{t\times r}$, $t = \frac{\text{S.I}\times 100}{p\times r}$ and

  $r = \frac{\text{S.I}\times 100}{p\times t}$
- **Amount (A) :** Total money paid by the borrower A = P + I or I = A – P
- **Rate (R) :** Interest on Rs. 100 for 1 year is known as the rate percent per annum.
- **Simple Interest (S.I):** Interest which is calculated uniformly on P throughout the loan period.
- **Compound Interest (C.I):** Interest obtained during the first time period is added to the original P and amount becomes new P for the second time period and so on. The difference between the amount obtained at the last itme period and original pricipal is called compound interest

$$A == P\left(1+\frac{R}{100}\right)^n \text{ or } \text{C.I.} = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$$

- **Conversion Period:** Fixed time period after which the interest is calculated and added to P to form the new P for the next time period

  If rates are different for different periods then,

  $$A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)..................$$

- **Growth:** Increase in the amount or anything over a period of time.

  $V_n = V_0\left(1+\frac{R}{100}\right)^n$, V_n = Value after growth in n conversions.

  V_o = Value in the begining.

  If the rate of growth varies for each conversion period then

  $$v_n = v_o\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)....$$

- **Depreciation :** Decrease in the amount or anything over a period of time

  $V_n = V_o\left(1-\frac{R}{100}\right)^n$, V_n = depreciated value after n conversion periods. V_0 = Value in the begining.

  If the rate of depreciation varies for each conversion period then

  $$V_n = V_o\left(1-\frac{R_1}{100}\right)\left(1-\frac{R_2}{100}\right)\left(1-\frac{R_3}{100}\right)....$$

- **Marked price or list price (M.P):** Price at which a article is listed for sale.
- **Discount:** Reduction in the marked price of an article.
- **Net selling price (S.P.):** SP = M.P – Discount

CHECK YOUR PROGRESS:

1. 0.0045 can be written, in percent, as:

   (A) 45% (B) 4.5% (C) 0.45% (D) 0.045%

2. In a fruit garden, there are 120 trees out of which 30 are mango trees. Percentage of other fruit trees in the garden is :

   (A) 25 (B) 30 (C) 70 (D) 75

3. What percent of the letters in the word ‘PERCENTAGE’ are E’s?

   (A) 10 (B) 20 (C) 30 (D) 40

4. Mohit purchased a watch for Rs. 1620 and spent Rs. 180 on its repair. If he sold it for Rs. 1980, then his gain percent is :

   (A) 19.8 (B) 16.2 (C) 18 (D) 10

5. Marked price of a rain coat is Rs. 450. If the shopkeeper sells it for Rs. 360, the discount given to the customer is :

   (A) 10% (B) 20% (C) 25% (D) 40%

6. A man sells two cows for Rs. 39600 each. On one he loses 10% while on the other , he gains 10%. Find the total loss or gain percent in the transaction.

7. The present cost of a machine is Rs. 4, 50, 000. In the first year its value depreciates at the rate of 10%. In second year by 8% and by 5% in the subsequent years. Find the worth of the machine at the end of 3 years.

8. In how much time will a sum of Rs. 8, 000 amount to Rs. 9261 at 10% per annum, compounded semi-aannually?
9. A sum of money amounts to Rs. 1680 in 2 years and to Rs. 1860 in 4 years at simple interest. Find the sum and the rate of interest per annum.
10. An article listed at Rs. 6800 is offered at a discoutn of 15%. Due to festival season, the shopkeeper allows a further discount of 5%. Find the selling price of the article.

STRETCH YOURSELF :

1. A watch was sold at a profit of 10%. Had it been sold for Rs. 35 more, the profit would have been 12%. Find the cost price of the watch.
2. If the cost price of 10 articles is equal to the selling price of 8 articles, then find the gain percent in this transaction.
3. A man bought bananas at 6 for Rs. 20 and sold at the rate of 4 for Rs. 18. Find the profit percent in this transaction.
4. A shopkeeper marks his goods 20% more than the cost price and allows a discount of 10%. Find the gain percent of the shopkeeper.
5. A reduction of 10% in the price of tea enables a dealer to buy 21 kg more tea for Rs. 2, 000. Find the reduced and original price of the tea per kg.

ANSWERS

CHECK YOUR PROGRESS:

1. C	2. D	3. C
4. D	5. B	6. Loss: 1%
7. Rs. 3, 53, 970		8. $1\frac{1}{2}$ years

9. Sum: Rs. 1500, rate of interest : 6%
10. Rs. 5491

STRETCH YOURSELF:

1. Rs. 1750
2. 25%
3. 35%
4. 8%
5. Reduced price/Kg = Rs. 135, Original Price/Kg = Rs. 150.

9

INSTALMENT BUYING

- **Cash Price:** Amount which a customer has to pay in full for the article at the time of purchase.
- **Cash down payment:** Partial payment made by a customer for an article at the time of purchase, under instalment plan.
- **Instalments :** Amount which is paid by the customer at regular intervals towards the remaining part of the S.P. (or cash price) of an article.
- **Interest under the instalment plan :** Under the instalment plan the buyer pays some extra amount which is interest on the deffered payment.

Under simple interest scheme S.I.. = $\frac{p\times r\times t}{100}$.

Under compount interest scheme C.I. = A – P

$$= P\left[\left(1+\frac{r}{100}\right)^n-1\right]$$

- **Types of word problems related to instalment scheme :** To find the rate of Interest

To find the amount of instalment

To find the cash price

Problems involving compound interest.

CHECK YOUR PROGRESS:

1. A dealer offers a micro wave oven for Rs. 5800 cash. A customer agrees to pay Rs. 1800 cash down and 3 equal instalments, the balance amount to be paid in equal instalments is:

   (A) Rs. 8000 (B) Rs. 6000 (C) Rs. 4000 (D) Rs. 2000

2. A watch is available for Rs. 970 cash or Rs. 350 cash down payment followed by 3 equal monthly instalments. If the rate of interest under this instalment plan is 24%. per annum. Find the amount of each instalment.

3. A cycle is availabe for Rs. 2700 cash or Rs. 600 cash down payment followed by 3 monthly instalments of Rs. 750 each. Find the rate of interest per annum charged under this instalment plan.

4. A mixi was purchased by paying Rs. 260 as cash down payment followed by three equal monthly instalments of Rs. 390 each. If the rate of interest charged under the instalment plan is 16% per annum, find the cash price of the mixi.

5. A washing machine is available for Rs. 15000 cash or Rs. 2000 cash down payment alongwith two eaual half yearly instalments. If the rate of interst charged under the instalment plan is 16% per annum compounded half yearly, find the amount of each instalment.

STRETCH YOURSELF:

1. A DVD player was purchased by a customer with a cash down payment of Rs. 2750 and agreed to pay 3 equal half yearly instalments of Rs. 331 each. If the interest charged was 20% per annum compounded half yearly, then find the cash price of the DVD player.
2. The selling price of a washing machine is Rs. 14000. The company asked for Rs. 7200 in advance and the rest to be paid in equal monthly instalments of Rs. 1400 each. If the rate of interest is 12% per annum, find the number of instalments.
3. A table is sold for Rs. 750 as cash down payment followed by Rs. 436 after a period of 6 months. If the rate of interest charged is 18% per annum, find the cash price of the table.

ANSWERS

CHECK YOUR PROGRESS:

1. C 2. Rs. 220 3. $44\frac{4}{9}\%$

4. Rs. 1500 5. Rs. 7290

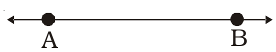
STRETCH YOURSELF:

1. Rs. 6060 2. 5 3. Rs. 1150

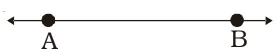
10

LINES AND ANGLES

- **Point:** A fine dot made by a sharp pencil on a sheet of paper. $\boxed{\cdot\ p}$
- **Line:** Fold a piece of paper, the crease in the paper represents a line. A line can be extended to any length on both sides. It has no end points. A line has no breadth and named using any two points on it i.e. $\overleftrightarrow{AB}$ or by a single small letter *l* or *m* or *n* etc.

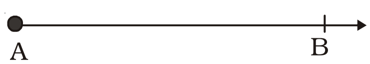


- **Line Segment :** The portion of the line between two points A and B is called a line segment and will be named $\overline{AB}$ or $\overline{BA}$. A line segment has two end points.

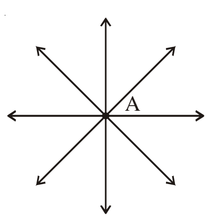


- **Ray :** A line segment AB when extended in one direction. It is denoted by $\overrightarrow{AB}$.

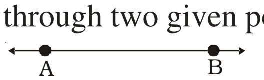
  Ray has one end point, called the initial point.

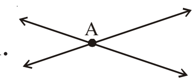


- **Plane :** A flat surface, which extends indefinitely in all directions e.g. surface of smooth wall, sheet of a paper etc.
- An infinite number of lines can be drawn through a point. All lines are called concurrent lines.

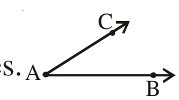


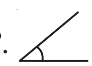
- One and only one line can be drawn passing through two given points.

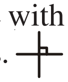


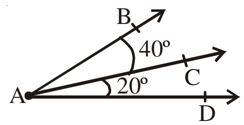
- If a line can pass through three or more points, then these points are said to be collinear otherwise points are non-collinear.
- Two distinct lines can not have more than one point in common. 

- Two lines in the same plane are called parallel lines if both have no points in common or if the distance between the lines is same every where

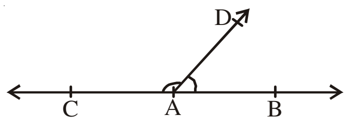
- Angle is formed by two rays with a common initial point called vertex and measured in degrees. 

- **Acute angle :** An angle whose measure is less than 90º. 
- **Right angle :** An angle whose measure is 90º. 
- **Obtuse angle :** An angle whose measure is more than 90º but less then 180º.

- **Straight angle :** An angle whose measure is 180º. 
- **Reflex angle :** An angle whose measure is more than 180º and less than 360^0. 
- Two lines or rays making a right angle with each other are called perpendicular lines. 
- **Complementary angles :** Two angles are said to be complementary to each other if the sum of their measures is 90º.
- **Supplementary angles :** Two angles are said to be supplementary if the sum of their measures is 180º.
- **Adjacent angles :** Two angles having a common vertex, a common arm and non common arms on opposite sides of the common arm. ∠BAC and ∠CAD are a "pair of adjacent angles".

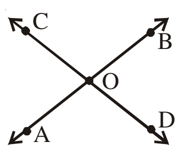


- **Linear Pair :** If AB and AC are opposite rays and AD is any other ray then ∠BAD and ∠CAD are said to form a linear pair.



- **Vertically opposite angles :** Two angles are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays.

  ∠AOC and ∠BOD, ∠AOD and ∠COB are pairs of vertically opposite angles.



- When a transversal intersects two parallel lines, then

  (i) each pair of corresponding angles are equal.

  (ii) each pair of alternate angles are equal.

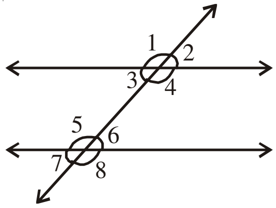
  (iii) each pair of interior angles on the same side of the transversal are supplementary.

  For example :

  (i) $\angle 2 = \angle 6$, $\angle 3 = \angle 7$

  (ii) $\angle 3 = \angle 6$ and $\angle 4 = \angle 5$

  (iii) $\angle 3 + \angle 5 = 180^\circ$ and $\angle 4 + \angle 6 = 180^\circ$



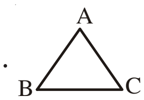
- When a transversal intersects two lines in such a way that

  (i) any pair of corresponding angles are equal

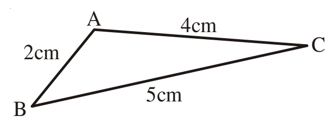
  or (ii) any pair of alternate angles are equal

  or (iii) any pair of interior angles on the same side of transversal are supplementary, then the two lines are parallel.

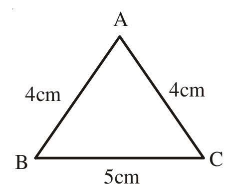
- **Triangle :** A plane figure bounded by three line segments.



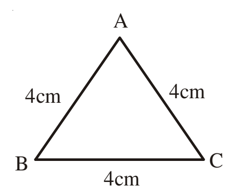
- **Scalene Triangle :** A triangle in which all the sides are of different lengths.



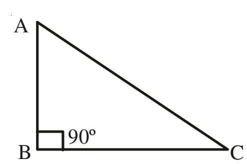
- **Isosceles Triangle :** A triangle having two sides equal.



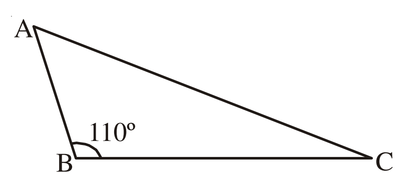
- **Equilateral Triangle :** A triangle having all sides equal.



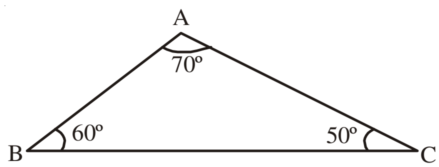
- **Right-angled Triangle :** A triangle in which one of the angles is right angle.



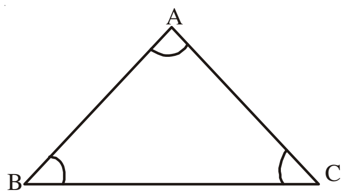
- **Obtuse angled triangle :** A triangle in which one of the angles is obtuse angle.



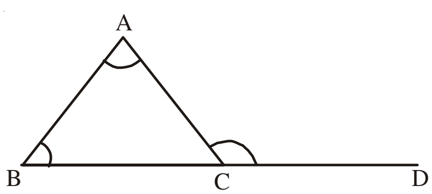
- **Acute angled triangle :** A triangle in which all the three angles are acute.



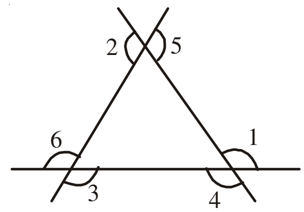
- The sum of the three interior angles of a triangle is 180º.
  ∠A + ∠B + ∠C = 180º



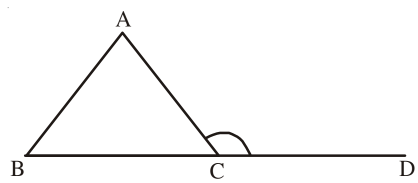
- The angle formed by a produced side of the triangle and another side of the triangle is called an exterior angle of the triangle.
  ∠ACD is an exterior angle.



or ∠1, ∠2, ∠3, ∠4, ∠5 and ∠6 all are exterior angles.



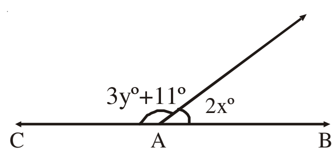
- Interior opposite angles are the angles of the triangle not forming a linear pair with the given exterior angle.
- An exterior angle of a triangle is equal to the sum of the two interior opposite angles.
  ∠ACD = ∠ABC + ∠BAC



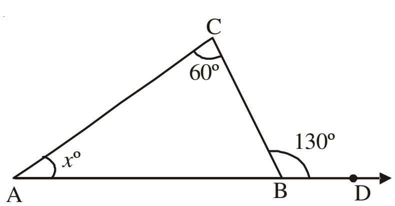
- Locus of a point moving under certain conditions is the path or the geometrical figure, every point of which satisfies the given conditions.
- The locus of a point equidistant from two given points is the perpendicular bisector of the line segment joining two points.
- The locus of a point equidistant from two intersecting lines is the pair of lines, bisecting the angles formed by the given lines.

CHECK YOUR PRGORESS

1. In figure AB and AC are opposite rays, if $x = 32^0$, then value of y is :
   A. 45^0 B. 32^0 C. 35^0 D. 105^0

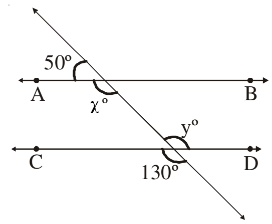


2. In given figure value of x is:



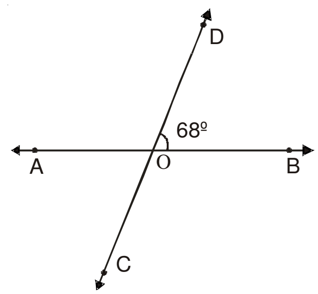
A. 45º B. 130º C. 30º D. 70º

3. In the figure, if AB || CD then values of x & y respectively are:



A. 130^0 , 130^0 B. 130^0, 50^0 C. 50^0, 130^0 D. 50^0, 50^0

4. In figure value of ∠COB is

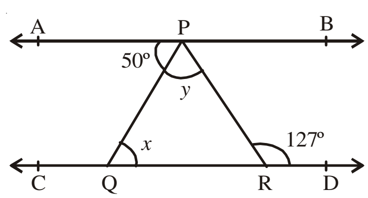


A. 36º B. 68º C. 112º D. 12º

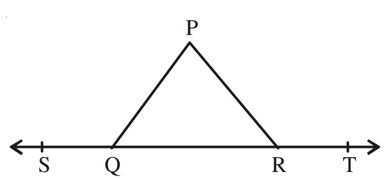
5. The angles of a triangle are in the ratio 1:2:3, the smallest angle of triangle is :

A. 30º B. 60º C. 90º D. 6º

6. In the fig., if AB || CD, ∠APQ = 50º and ∠PRD = 127º, find x any y.

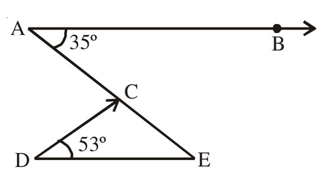


7. In fig. if ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT.

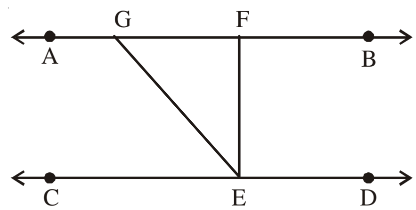


8. Prove that the sum of all interior angles of a triangle is 180º.

9. In fig. if AB || DE, ∠BAC = 35º and ∠CDE = 53º find ∠DCE.

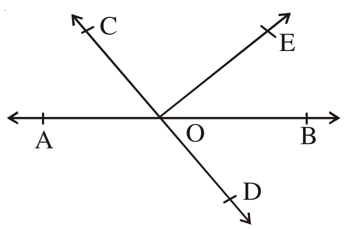


10. In fig., if AB || CD, EF ⊥ CD and ∠GED = 126º find ∠AGE, ∠GEF and ∠FGE.

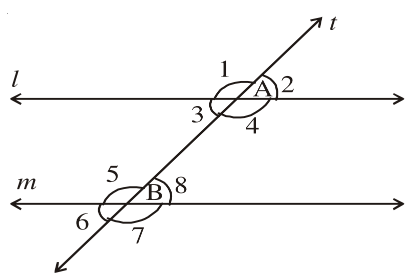


STRETCH YOURSELF

1. In fig., lines AB and CD intersect at O. If ∠AOC + ∠BOE = 70º and ∠BOD = 40º find ∠BOE and reflex ∠COE.



2. In fig. *l* || *m* and transversal '*t*' intersects *l* and *m* at A and B respectively. If ∠1 : ∠2 = 3 : 2, determine all the eight angles.



ANSWERS

CHECK YOUR PROGRESS :

1. C
2. D
3. A
4. C
5. A
6. $x = 50^0$, $y = 77^0$
9. 92^0
10. $\angle AGE = 126^0$, $\angle GEF = 36^0$, $\angle FGE = 54^0$

STRETCH YOURSELF :

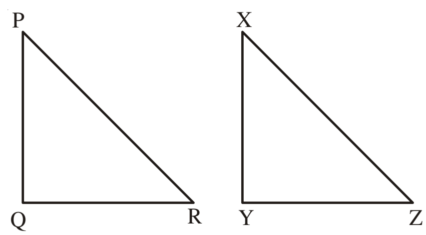
1. $\angle BOE = 30^0$, Reflex $\angle COE = 250^0$
2. ∠1 = 108º, ∠2 = 72º, ∠3 = 72º $\angle 4 = 108^0$, $\angle 5 = 108^0$, $\angle 6 = 72^0$, $\angle 7 = 108^0$, $\angle 8 = 72^0$

11

CONGRUENCE OF TRIANGLES

- Two figures, which have the same shape and same size are called congruent figures and this property is called congruence.
- Two line segments are congruent when they are of equal length.
- Two squares are congruent if their sides are equal.
- Two triangles are congruent, if all the sides and all the angles of one are equal to the corresponding sides and angles of other.

For example in triangles PQR and XYZ



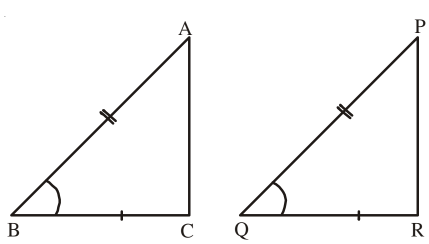
PQ = XY, PR = XZ, QR = YZ

$\angle P = \angle X$, $\angle Q = \angle Y$, $\angle R = \angle Z$

Thus ΔPQR is congruent to ΔXYZ and we write $\Delta PQR \cong \Delta XYZ$ where $\cong$ is symbol of congruence.

- **SAS Criterion of congruence:** If the two sides and the included angle of one triangle are equal to the corresponding sides and included angle of the other triangle, the two triangles are congruent.

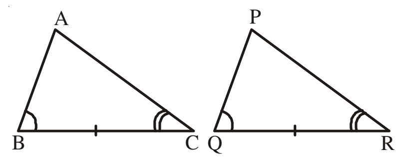
For example:



AB = PQ, BC = QR, $\angle ABC = \angle PQR$

Hence $\Delta ABC \cong \Delta PQR$

- **ASA or AAS Criterion of Congruence:-** If any two angles and one side of a triangle are equal to corresponding angles and the side of the another triangle, then the two triangles are congruent.
  For example

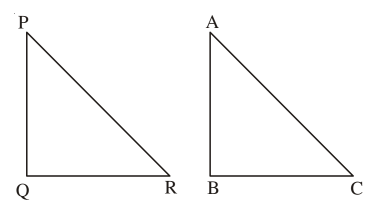


$\angle ABC = \angle PQR$, $\angle ACB = \angle PRQ$ and BC = QR

Hence $\Delta ABC \cong \Delta PQR$

- **SSS Criterion of Congruence:** If the three sides of one triangle are equal to the corresponding sides of another triangle, then the two triangles are congruent.

For example:

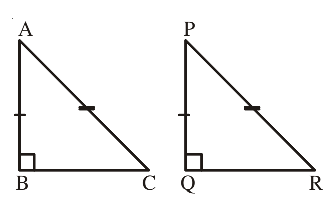


AB = PQ, BC= QR, AC= PR,

Hence $\Delta ABC \cong \Delta PQR$

- **RHS Criterion of Congruence:** If the hypotenuse and a side of one right triangle are respectively equal to the hypotenuse and a side of another right tringle, then the two triangles are congruent.

For example



AC= PR, AB= PQ, $\angle ABC = \angle PQR = 90^{\circ}$

Hence $\Delta ABC \cong \Delta PQR$

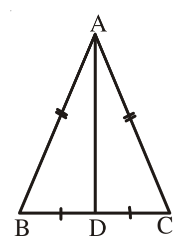
- The angles opposite to equal sides of a triangle are equal.
- The sides opposite to equal angles of a triangle are equal.
- Perpendiculars or altitudes drawn on equal sides, from opposite vertices of an isosceles triangle are equal.
- If two sides of a triangle are unequal, then the longer side has the greater angle opposite to it.
- In a triangle, the greater angle has longer side opposite to it.
- Sum of any two sides of a triangle is greater than the third side.

CHECK YOUR PROGRESS:

1. In triangle ABC if $\angle C > \angle B$, then :

   A. BC > AC B. AB > AC C. AB < AC D. BC < AC

2. In figure if AB = AC and BD = DC, then $\angle ADB$ is :



   A. 45° B. 90° C. 60° D. None of these

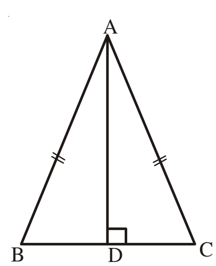
3. Two sides of a triangle are of length 6 cm and 2.5 cm. The length of the third side of the triangle can not be :

   A. 4.5 cm B. 5 cm C. 6 cm D. 3.2 cm

4. In ΔPQR, QR = PQ and $\angle Q = 40^0$, then $\angle P$ is equal to -

   A. 40^0 B. 70^0 C. 50^0 D. 80^0

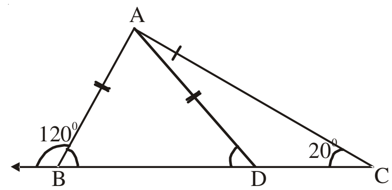
5. In ΔABC, if $\angle B = \angle C$ and AD $\perp$ BC, then $\Delta ABD \cong \Delta ACD$ by the criterian:



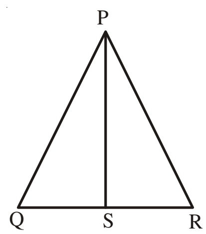
   A. RHS B. ASA C. SAS D. SSS

6. Δ ABC is a right triangle in which $\angle B = 90°$ and AB = BC. Find $\angle A$ and $\angle C$.

7. In figure. Find $\angle DAC$

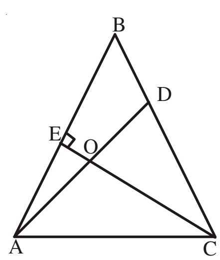


8. Prove that angles opposite to equal sides of a triangle are equal.
9. Prove that each angle of an equilateral triangle is 60^0.
10. S is any point on side QR of a Δ PQR. Show that PQ + QR + RP > 2PS



STRETCH YOURSELF

1. Show that in a quadrilateral ABCD, AB + BC + CD + DA > AC+ BD.
2. A triangle ABC is right angled at A. AL is drawn perpendicular to BC. Prove that $\angle BAL = \angle ACB$.
3. Prove that the medians of a triangle are equal.
4. In figure $\angle A = \angle C$ and AB=AC. Prove that $\Delta ABD \cong \Delta CBE$.



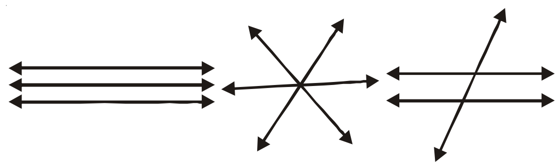
ANSWERS:

CHECK YOUR PROGRESS :

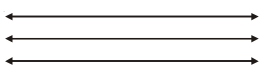
1. B
2. B
3. D
4. B
5. A
6. $\angle A = 45^0$, $\angle B = 45^0$
7. $\angle DAC = 40^0$

12
CONCURRENT LINES

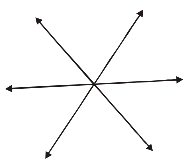
- Two lines in a plane can either be parlallel or intersecting.



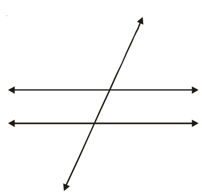
Three lines in a plane may:
(i) be parallel to each other



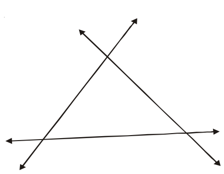
(ii) intersect each other in exactly one point



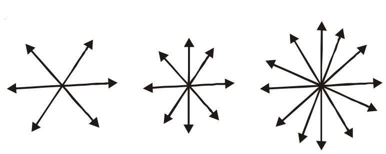
(iii) intersect each other in two points



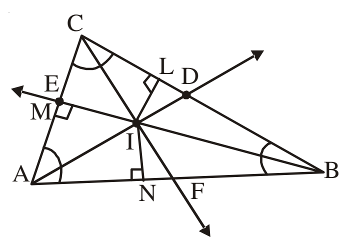
(iv) intersect each other at most in three points



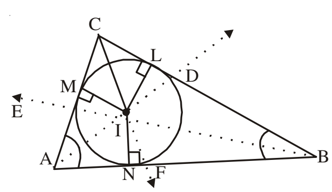
- Three or more lines in a plane which intersect each other in exactly one point or pass through the same point are called concurrent lines and the common point is called the point of concurrency.



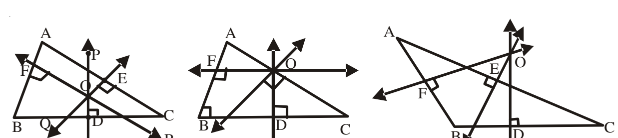
- A line which bisects an angle of a triangle is called an angle bisector of the triangle.
- A triangle has three angle bisectors in it.
- Angle bisectors of a triangle pass through the same point.
- AD, BE and CF are three angle bisectors of $\triangle$ABC which passes through same point I. I is called incentre of the triangle.



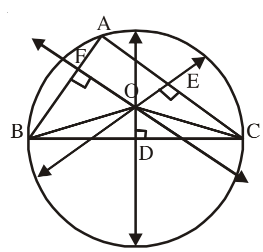
- Incentre always lies in the interior of the triangle and at the same distance from the three sides of the triangle i.e. IL = IM = IN
- If we take I as centre and IL or IM or IN as radius and draw a circle then the circle is called "incircle" of the triangle.



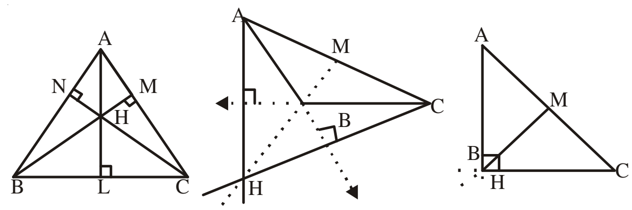
- A line which bisects a side of a trinagle at right angle is called the perpedicular bisector of the side.
- The three perpendicular bisectors of the sides of a triangle pass throught the same point. The point of concurrency O is called the 'circumcentre' of the triangle.



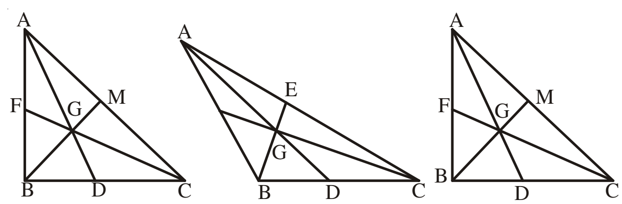
- **Circumcentre will be**
  (i) In the interior of the triangle for an a acute triangle.
  (ii) On the hypotensue of a right angle.
  (iii) in the exterior of the triangle for an obtuse triangle.
- All the three perpendicular bisectors of a triangle pass through O and it is called circumcentre which is equidistant from vertices A, B and C.
- If we mark O as centre and OA or OB or OC as radius and draw a circle. The circle passes through A, B and C of the traingle called 'circumcircle' of the triangle.



- Perpendicular drawn from a vertex of a triangle on the opposite side is called its altitude.
- In a triangle the three altitudes pass through the same point and the point of concurrency is called the 'orthocentre' of the triangle.
- **Orthocentre will be**
  (i) In the interior of the triangle for an acute triangle.
  (ii) At the vertex containing the right angle for a right triangle.
  (iii) In the exterior of the triangle for an obtuse triangle.



- A line joining a vertex to the mid point of the opposite side of a triangle is called its median.
- All the three medians pass through the same point. The point of concurrency 'G' is called the centroid of the triangle.

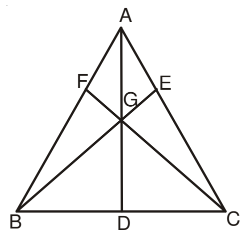


- Centroid divides each of the medians in the ratio 2:1.
- In an isosceles triangle, bisector of the angle formed by the equal sides is also a perpendicular bisector, an altitude and a median of the triangle.
- In an equilateral triangle the angle bisectors are also the perpendicular bisectors of the sides, altitudes and medians of the triangle.

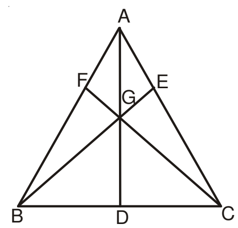
CHECK YOUR PROGRESS:

1. In a plane the point equidistant from vertices of a triangle is called its-
   A. Centroid B. Incentre C. Circumcentre D. Orthocentre
2. In a plane the point equidistant from the sides of the triangle is called its-
   A. Centroid B. Incentre C. Circumcentre D. Orthocentre
3. Centroid of a triangle divides median in the ratio-
   A. 2 : 1 B. 1 : 2 C. 1:1 D. None of these
4. The incentre of the triangle always lies in the
   A. Exterior of the triangle B. On the triangle
   C. Interior of the triangle D. None of these

5. Three or more lines in a plane which intersect each other in exactly one point or which pass through the same point are called:
   A. Parallel lines B. Concurrent lines
   C. Congruent lines D. Bisectors
6. In an equilateral ΔABC, G is the centroid, If AG is 7.2cm, find AD and BE.



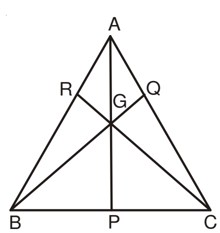
7. In figure if AD= 4.8 cm, D is the mid point of BC. Find AG.



8. In an equilateral triangle show that the incentre, the circumcentre, the orthocentre and the centroid are the same point.
9. ABC is an equilateral triangle of side 24cm. If G be its centroid. Find AG..
10. ABC is an isosceles triangle such that AB = AC = 10cm and base BC= 8cm. If G is the centroid of Δ ABC, find AG.

STRETCH YOURSELF

1. Find the circumradius of cirumcircle and inradius of incircle of an equilateral triangle of side 2a.
2. In an equilateral Δ ABC, if G is centroid and AG = 6cm find the side of the triangle.
3. In an isosceles triangle, show that the bisector of the angle formed by the equal sides is also a perpendicular bisector, altitude and a median of the triangle.
4. In figure P, Q and R are the mid-points of the sides of Δ ABC. Show that



$BQ - CR > \frac{3}{2}BC.$

5. If O is the orthocentre of ΔPQR, then show that P is the orthocentre of the ΔOQR.

ANSWER:

CHECK YOUR PROGRESS :

1. C
2. B
3. A
4. C
5. B
6. AD = 10.8cm, BE = 10.8cm
7 AG = 3.2cm
9. $8\sqrt{3}$ cm
10. AG = 4cm

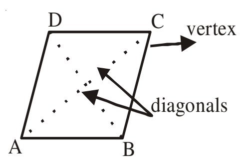
STRETCH YOURSELF :

1. Circumradius = $\frac{2a}{\sqrt{3}}$, inradius = $\frac{a}{\sqrt{3}}$
2. $6\sqrt{3}$ cm

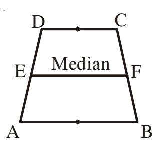
13

QUADRILATERALS

- ***Quadrilateral:-*** A plane, closed, geometric figure with four sides.

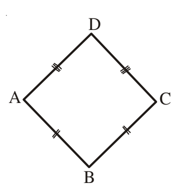


- **Elements of a Quadrilateral.**
  Four sides- AB, BC, CD and DA
  Four angles- $\angle A, \angle B, \angle C, \angle D$
  Two diagonals- AC and BD
  Four vertices- A, B, C and D
- **Types of Quadrilaterals**
- **Trapezium:** When one pair of oppsite sides of quadrilateral is parallel, then it is called a trapezium.

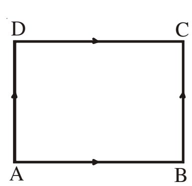


In figure. AB || DC, AB and DC are called bases of the trapezium.
If non-parallel sides of a trapezium are equal, then it is called an isosceles trapezium.

- **Kite**: When two pairs of adjacent sides of a quadrilateral are equal, then it is called a kite.

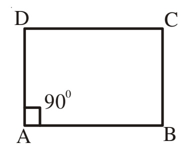


- **Parallelogram:** When both the pairs of opposite sides of a quadrilateral are parallel, then it is called a parallelogram.

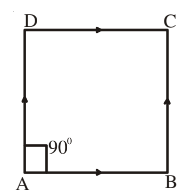


AB || DC and AD || BC

- **Rectangle** : It is a special type of parallelogram when one of its angles is right angle.



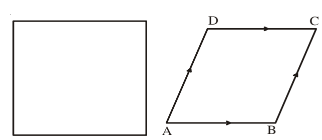
- **Square:** When all the four sides of a parallelogram are equal and one of its angles is 90^0, then it is called a square.



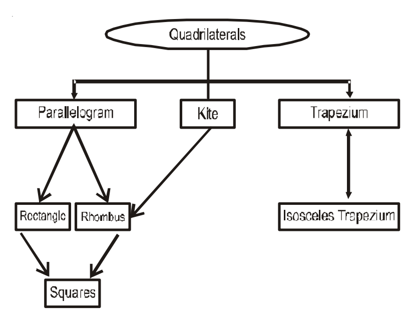
In □ ABCD

AB = BC = CD = DA and $\angle A = 90^0$.

- **Rhombus** : When all four sides of a parallelogram are equal, then it is called a rhombus.



- **Types of quadrilaterls**

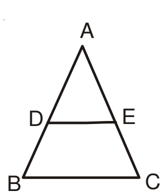


Properties of different types of quadrilaterals:

1. Parallelogram	The opposite sides are equal. The opposite angles are equal. The diagonals bisect each other and each of them divides the parallelogram into two triangles of equal area.
2. Rhombus	All sides are equal. Opposite angles are equal Diagonals of a rhombus are unequal and bisect each other at right angles.
3. Rectangle	Opposite sides are equal. Each angle is a right angle. Diagonals are equal and bisect each other.
4. Square	All sides are equal. Each of the angles measures 90^0. Diagonals are equal and bisect each other at right angles.

Mid-Point Theorem:

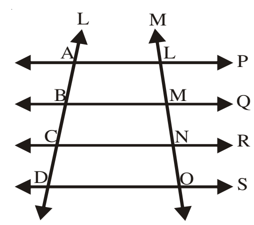
- In a triangle the line-segment joining the mid points of any two sides is parallel to the third side and is half of it.



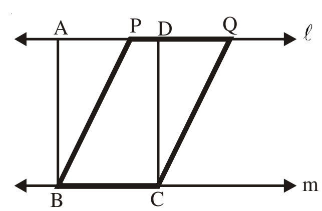
In ΔABC if D and E are the mid-points of AB and AC respectively then DE || BC and

$DE = \frac{1}{2}BC$.

- The line drawn through the mid point of one side of a triangle parallel to the another side, bisects the thrid side.
- If there are three or more parallel lines and the intercepts made by them on a transversal are equal, the corresponding intercepts made on any other transversal are also equal e.g. if AB = BC = CD then LM = MN = NO.



- Parallelograms on the same base (or equal bases) and between the same parallels are equal in the area. If l || m then

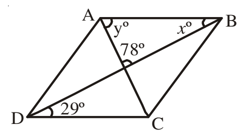


area of || gram ABCD = area of || gram PBCQ

- Triangles on the same base (or equal bases) and between the same parallels are equal in area.
- Triangles on equal bases having equal areas have their corresponding altitudes equal.

CHECK YOUR PROGRESS:

1. In parallelogram, ABCD find the value of x and y-

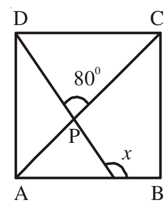


A. 29^0, 73^0 B. 23^0, 78^0 C. 23^0, 23^0 D. 29^0, 78^0

2. Three angles of a quadrilateral measure 54^0, 110^0 and 86^0. The measure of the fourth angle is:

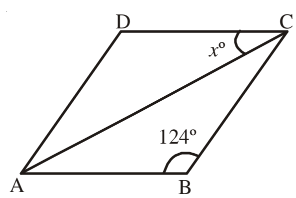
A. 86^0 B. 54^0 C. 110^0 D. 250^0

3.. In figure, ABCD is a square. If $\angle DPC = 80^0$, then value of x is



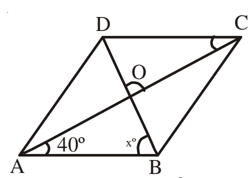
A. 125° B. 130° C. 120° D. 115°

4. In figure ABCD is a rhombus. If $\angle ABC$ is 124°, then the value of x is



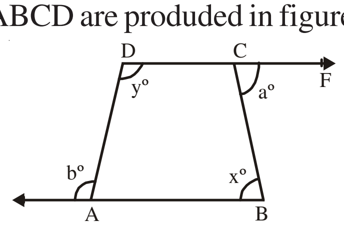
A. 26° B. 28° C. 25° D. 27°

5. In figure ABCD is a rhombus whose diagonals intersect at O. If $\angle OAB = 40°$ and $\angle ABO = x°$, then x = ?



A. 50^0 B. 35^0 C. 40^0 D. 45^0

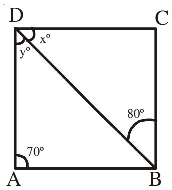
6. The length of the diagonals of a rhombus are 24 cm and 18 cm respectively. Find the length of each side of the rhombus.
7. Prove that the sum of all the four angles of a quadrilateral is 360^0.
8. The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.
9. The sides BA and DC of □ ABCD are produced in figure. Prove that x + y = a+b.



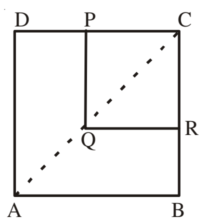
10. Show that the diagonals of a square are equal and bisect each other at right angles.

STRETCH YOURSELF

1. In figure ABCD is a parallelogram in which $\angle DAB = 70^\circ$, $\angle DBC = 80^\circ$. Find x and y



2. ABCD and PQRC are rectangles where Q is the mid point of AC. Prove that (i) DP= PC



(ii) $PR = \frac{1}{2} AC$

3. If D, E and F are the mid-points of the sides BC, CA and AB respectively of an equilateral triangle ABC. Prove that ΔDEF is also an equilateral triangle.

4. ABC is a triangle right angled at C. A line through the mid point M of hypotenuse AB and parallel to BC intersects AC at D. Show that
(i) D is the mid point of AC.
(ii) $MD \perp AC$
(ii) $CM = AM = \frac{1}{2} AB$

5. Prove that the line segment joining the mid points of any two sides of a triangle is parallel to the third side and equal to half of it.

ANSWER

CHECK YOUR PROGRESS :

1. A
2. C
3. A
4. B
5. A
6. 15cm
8. 36^0, 60^0, 108^0, 156^0

STRETCH YOURSELF :

1. $x = 30^0$, $y = 80^0$

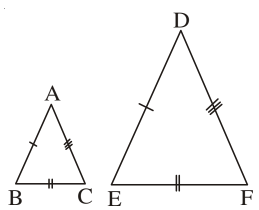
14

SIMILARITY OF TRIANGLES

- Objects which have the same shape but different sizes are called similar objects.
- Any two polygons, with corresponding angles equal and corresponding sides proportional are similar.
- Two triangles are similar if

  (i) their corresponding angles are equal and

  (ii) their corresponding sides are proportional



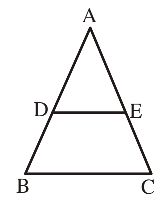
$\Delta ABC \sim \Delta DEF$ if $\angle A = \angle D$, $\angle B = \angle E$,

$\angle C = \angle F$ and $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{CA}{FD}$

The symbol ' ~ ' stands for "is similar to".

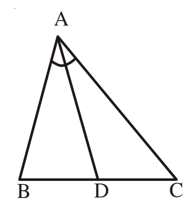
- **AAA Criterion for Similarity:** If in two triangles the corresponding angles are equal, the triangles are similar.
- **SSS Criterion for Similarity:** If the corresponding sides of two triangles are proportional, the triangles are similar.
- **SAS Criterion for Similarity:** If one angle of a triangle is equal to one angle of the other triangle and the sides containing these angles are proportional, the triangles are similar.
- If a line drawn parallel to one side of a triangle intersects the other two sides at distinct points, the other two sides of the triangles are divided proportionally.

If DE || BC then $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

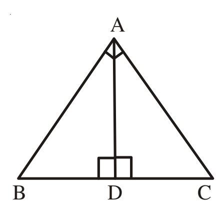


- If a line divides any two sides of a triangle in the same ratio, the line is parallel to third side of the triangle.
- The internal bisector of any angle of a triangle divides the opposite side in the ratio of sides containing the angle. If AD is internal bisector of $\angle A$ of ΔABC, then

$$\frac{BD}{DC} = \frac{AB}{AC}$$



- If a perpendicular is drawn from the vertex containing right angle of a right tringle to the hypotenuse, the triangles on each side of the perpendicular are similar to each other and to the original triangle.



$\Delta ADB \sim \Delta CDA$, $\Delta ADB \sim \Delta CAB$ and $\Delta ADC \sim \Delta BAC$..

- The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

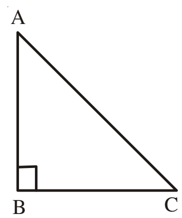
$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

- **Baudhayan/Pythagoras Theorem**

  In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

  In triangle ABC

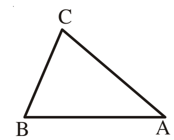
  $AC^2 = AB^2 + BC^2$



- **Converse of pythagoras Theorem**

  In a triangle, if the square on one side is equal to the sum of the squares on the other two sides, the angle opposite to the first side is a right angle.

  If in triange ABC,



$AC^2 = AB^2 + BC^2$

then $\angle B = 90^0$

CHECK YOUR PROGRESS:

1. The areas of two similar triangles are 25 sq. m and 121 sq. m. The ratio of their corresponding sides is :

   (A) 5 : 11 (B) 11 : 5 (C) $\sqrt{5} : \sqrt{11}$ (D) $\sqrt{11} : \sqrt{5}$

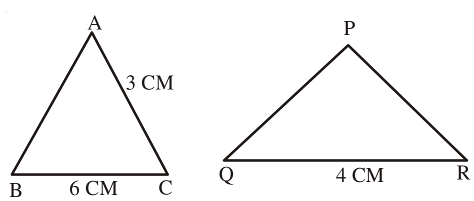
2. Two poles 6m and 11m high stands vertically on the ground If the distance between their feet is 12m, then the distance between their tops is:

   (A) 11m (B) 12 m (C) 13 m (D) 14m

3. If in two triangles DEF and PQR, $\angle D = \angle Q$, $\angle R = \angle E$, which of the following is not true?

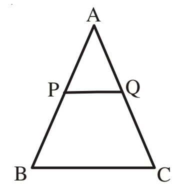
   (A) $\frac{DE}{PQ} = \frac{EF}{RP}$ (B) $\frac{EF}{PR} = \frac{DF}{PQ}$ (C) $\frac{DE}{QR} = \frac{DF}{PQ}$ (D) $\frac{EF}{RP} = \frac{DE}{QR}$

4. In the adjoining figure, $\Delta ABC \sim \Delta PQR$, length of PR is:



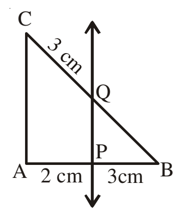
   (A) 3cm (B) 2cm (C) 4cm (D) 6cm

5. In the adjoining figure, P and Q are mid points of AB and AC respectively, If PQ = 3.4cm , then BC is :



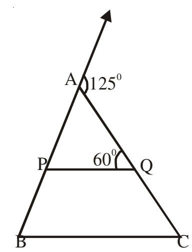
(A) 3.4cm (B) 1.7 cm (C) 6.8cm (D) 10.2cm

6. In the adjoining figure, QP || CA, find BC

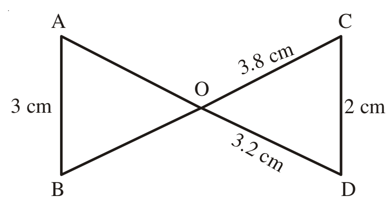


7. In ΔABC if AB= a cm, BC= $\sqrt{3}$ a cm and AC= 2a cm, then find ∠B

8. In the adjoining figure ΔABC ~ ΔAPQ. Find ∠B



9. In the adjoining figure ΔABO ~ ΔDCO. Find OA and OB



10. In an equilateral ΔABC, AD ⊥ BC Prove that 3 $AB^2 = 4\ AD^2$.

STRETCH YOURSELF

1. Show that the altitude of an equilateral triangle with side a is $\frac{\sqrt{3}}{2}a$.
2. In ΔABC, AD ⊥ BC and $AD^2 = BD \times DC$. Prove that ∠BAC= 90^0.
3. Prove that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.
4. If a line is drawn parallel to one side of a triangle intersecting the other two sides, then prove that the line divides the two sides in the same ratio.
5. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.

ANSWERS:

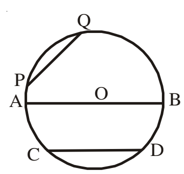
CHECK YOUR PROGRESS :

1. A
2. C
3. A
4. B
5. C
6. 7.5cm
7. 90^0
8. 65^0
9. 4.8cm
10. 5.7cm

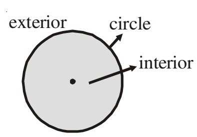
15

CIRCLE

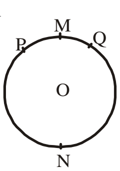
- A circle is a collection of all points in a plane which are at a constant distance from a fixed point. The fixed point is called the centre of the circle.
- A line segment joining centre of the circle to a point on the circle is called radius of the circle. The circle has infinite no. of radii. All radii of a circle are equal.
- A line segment joining any two points on the circle is called a chord. Chord passing through the centre of circle is called its diameter. Diameter is the longest chord of the circle.



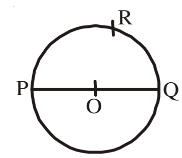
- Shaded region is interior, the boundary is circle and unshaded region is exterior of the circle.



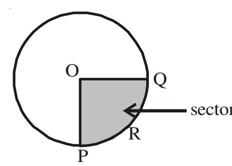
- **Arc:** A part of a circle. Here PMQ is an arc denoted by $\overset{\frown}{PMQ}$.



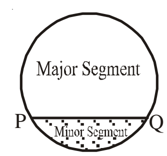
- **Minor arc:** An arc of a circle whose length is less than that of a semi-circle of the same circle. PMQ is a minor arc.
- **Major arc:** An arc of a circle whose length is greater than that of a semi circle of the same circle is called a major arc. PNQ is a major arc.
- Diameter of a circle divides a circle into two equal arcs, each of which is called a semi circle. In figure $\overset{\frown}{PRQ}$ is semi-circle.



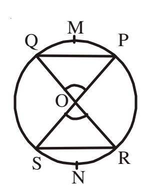
- **Sector:** The region bounded by an arc of a circle and two radii.



- **Segment:** A chord divides the interior of a circle into two parts. Each of which is called a segment.

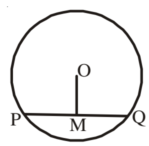


- **Circumference:** The length of the boundary of a circle is the circumference of the circle. The ratio of the circumference of circle to its diameter is always a constant, which is denoted by Greek letter π.
- Two arcs of a circle are congruent if and only if the angles subtended by them at the centre are equal, arc PMQ $\cong$ arc SNR $\Leftrightarrow \angle POQ = \angle SOR$.



- Two arcs of a circle are congruent if and only if their corresponding chords are equal, arc QMP $\cong$ arc SNR $\Leftrightarrow$ PQ = RS.

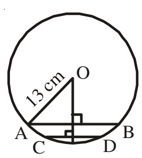
- Equal chords of a circle subtend equal angles at the centre and conversely if the angles subtended by the chords at the centre of a circle are equal, then the chords are equal.
- The perpendicular drawn from the centre of a circle to a chord bisects the chord. $OM \perp PQ \Rightarrow PM = MQ$.



- Conversely the line joinning the centre of a circle to the mid-point of a chord is perpendicular to the chord.
- There is one and only one circle passing through three non-collinear points.
- Equal chords of a circle are equidistant from the centre, conversely chords that are equidistant from the centre of a circle are equal.

CHECK YOUR PROGRESS:

1. In figure given below, AB = 8cm and CD = 6 cm are two parallel chords of a circle with centre O. Distance between the chords is

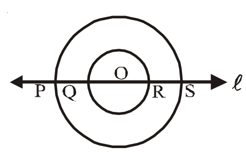


(A) 2 cm (B) 1 cm (C) 1.5 cm (D) 3 cm

2. A regular octagon is inscribed in a circle. The angle subtended by each side of octagon at the centre of circle is

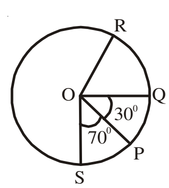
(A) 72º (B) 45º (C) 74º (D) 66º

3. In figure a line l intersects the two concentric circles with centre O at points P, Q, R and S then



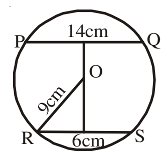
(A) PQ + RS = OQ + OR (B) OP = 2OQ
(C) OS – RS = OP – OQ (D) PQ = RS

4. In figure given below arc PQ ≅ arc QR, $\angle POQ = 30^\circ$ and $\angle POS = 70^\circ$ then $\angle ROS$ is



(A) 200º (B) 150º (C) 230º (D) 120º

5. In figure PQ = 14cm and RS = 6cm are two parallel chords of a circle with centre O. Distance between the chords PQ and RS is



(A) $6\sqrt{2}$ cm (B) $10\sqrt{2}$ cm (C) $4\sqrt{2}$ cm (D) $2\sqrt{2}$ cm

6. Two circles with centres O and O' intersect at the points A and B. Prove that $\angle OAO' = \angle OBO'$.

7. If two equal chords of a cricle intersect inside the circle, then, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

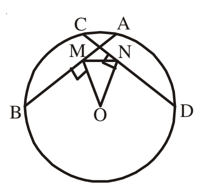
8. Two chords AB and AC of a circle are equal. Prove that the centre of the circle is on the angle bisector of $\angle BAC$.

9. If two circles intersect at two points. Prove that their centres are on the perpendicular bisector of the common chord.

10. AB and CD are two parallel chords of a circle which are on opposite sides of the centre such that AB= 10cm, CD= 24cm and the distance between AB and CD is 17cm. Find the radius of the circle.

STRETCH YOURSELF

1. In figure given below AB and CD are two equal chords of a circle whose centre is O. OM $\perp$ AB and ON $\perp$ CD. Prove that $\angle OMN = \angle ONM$.



2. A circle with centre O has chords AB and AC such that AB= AC=6cm. If radius of circle is 5cm, then find the length of chord BC.

3. Two circles with centres O and O' intersect at point P. A line l is drawn through point P parallel to OO' which intersects them at the points C and D. Prove that CD = 2 × OO'.

ANSWERS:

CHECK YOUR PROGRESS :

1. B
2. B
3. D
4. D
5. B

10. 5.13cm

STRETCH YOURSELF :

2. 9.6cm

16

ANGLES IN A CIRCLE AND CYCLIC QUADRILATERAL

- **Central Angle:** Angle subtended by an arc at the centre of circle.

  In figure it is $\angle$AOB.

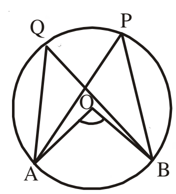


Fig. (i)

Length of an Arc =

$$\text{circumference} \times \frac{\text{degree measure of the arc}}{360^0}$$

- **Inscribed Angle :** The angle subtended by an arc or chord on any point on the remaining part of circle . In figure (i) it is $\angle$APB.

  The angle subtended at the centre of a circle by an arc is double the angle subtended by it on any point on the remaining part of the circle. In fig. (i) $\angle AOB = 2\angle APB$.

  Angles in the same segment of a circle are equal . In fig. (i) $\angle APB = \angle AQB$.

- Angle in a semi circle is a right angle.

  In Fig. (ii) $\angle PBQ = 90^o$

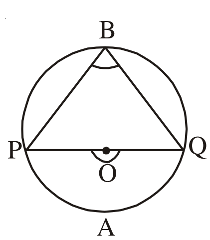


Fig. (ii)

- **Concyclic Points:** Points which lie on a circle

  Three non collinear points are always concylcic and a uniuqe circle passes through them

- **Cyclic Quadrilateral :** A quadrilateral in which all four vertices lie on a circle. In fig. (iii) PQRS is a cyclic quadrilateral.

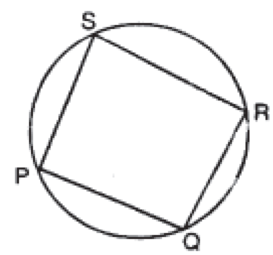


Fig. (iii)

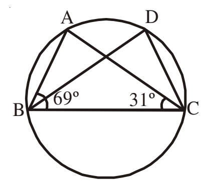
If a pair of opposite angles of a quadrilateral is supplementary then the quadrilateral is cyclic i.e. $\angle P + \angle R = 180^0$ or

$\angle Q + \angle S = 180^0 \Rightarrow$ PQRS is cyclic.

- If PQRS is a cyclic parallelogram then it is a rectangle.

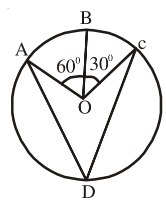
CHECK YOUR PROGRESS:

1. In given figure if $\angle ABC = 69^\circ$ and $\angle ACB = 31^\circ$ then $\angle BDC$ is:



(A) 80° (B) 69° (C) 59° (D) 31°

2. In figure given below A, B and C are three points on a circle with centre O such that $\angle BOC = 30^\circ$ and $\angle AOB = 60^\circ$. If D is a point on the circle other than the arc ABC, then $\angle ADC$ is:

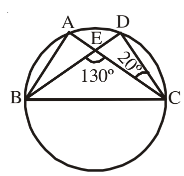


(A) 30° (B) 60° (C) 45° (D) 90°

3. A chord of a circle is equal to the radius of the circle. The angle subtended by the chord at a point on the minor arc is :

(A) 15° (B) 150° (c) 45° (D) 60°

4. In figure given below A, B, C and D are four points on a circle. ACand BD intersect at a point E such that $\angle BEC = 130^\circ$ and $\angle ECD = 20^0$. $\angle BAC$ is :

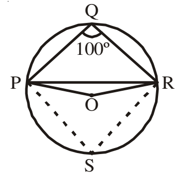


(A) 110° (B) 60° (C) 120° (D) 90°

5. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^0$, $\angle BAC = 30^0$, find $\angle BCD$. Further if AB = BC, find $\angle ECD$.

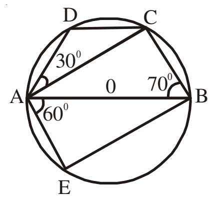
(A) 30° (B) 60° (C) 50° (D) 110°

6. In given figure $\angle PQR = 100^\circ$, where P, Q and R are the points on a circle with centre O. $\angle OPR$ is :

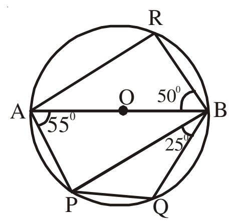


(A) 70^0 (B) 80^0 (C) 10^0 (D) 20^0

7. In given figure AB is a diameter of a circle with centre O. If $\angle ABC = 70^0$, $\angle CAD = 30^0$ and $\angle BAE = 60^0$, find $\angle BAC$, $\angle ACD$ and $\angle ABE$.

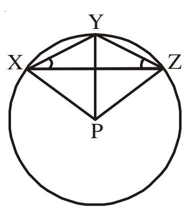


8. In figure AB is the diameter of a circle with centre O. If $\angle PAB = 55^o$, $\angle PBQ = 25^o$ and $\angle ABR = 50^o$, then find $\angle PBA$, $\angle BPQ$ and $\angle BAR$



STRETCH YOURSELF

1. In figure given below P is the centre of a circle. Prove that $\angle XPZ = 2\,(\angle XZY + \angle YXZ)$.



2. Two circles intersect at A and B. AC and AD are diameters of the circles. Prove that C, B and D are collinear.

ANSWERS

CHECK YOUR PROGRESS :

1. A

2. C

3. B

4. A

5. C

6. C

7. 20º, 40º, 30º

8. 35º, 30º, 40º

17

SECANTS, TANGENTS AND THEIR PROPERTIES

- **Secant :** A line which interesects circle at two distinct points. Here PAB is a secant.

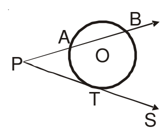


Fig. (i)

- **Tangent :** A line which touches a circle at exactly one point and the point where it touches the circle is called point of contact. Here PTS is tangent and T is point of contact.

  When two points of intersection of secant and circle coincide it becomes a tangent.

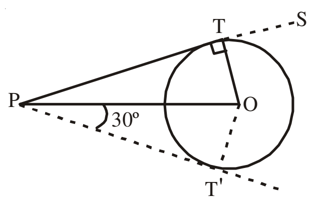


Fig. (ii)

- From an external point only two tangents can be drawn to a circle e. g. PT & PT′.
- The lengths of two tangents from an external point are equal. Here PT = PT′, [Fig. (ii)]
- A radius through the point of contact is perpendicular to the tangent at the point. Here ∠PT′ O = ∠PTO = 90º. [Fig. (ii)]
- The tangents drawn from an external point to a circle are equally inclined to the line joining the point to the centre of circle. Here ∠TPO = ∠T′PO. [Fig. (ii)]

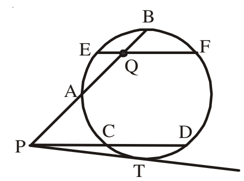


Fig. (iii)

If two chords AB and CD or AB and EF of a circle intersect at a point P or Q outside or inside the circle, then PA × PB = PC × PD or QA × QB = QE × QF.

If PAB is a secant to a circle intersecting the circle at A and B and PT is a tangent to the circle at T, then PA × PB = PT^2. [Fig. (i)]

The angles made by a chord in alternate segment through the point of contact of a tangent is equal to the angle between chord and tangent. Here ∠QP X = ∠QSP and ∠PRQ = ∠QPY. [Fig. (iv)]

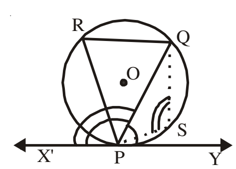


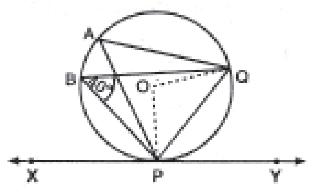
Fig. (iv)

CHECK YOUR PROGRESS:

1. A circle touches all the four sides of a quadrilateral ABCD. Prove that AB + CD = BC + DA.
2. Prove that a parallelogram circumscribing a circle is a rhombus.
3. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2∠OPQ.

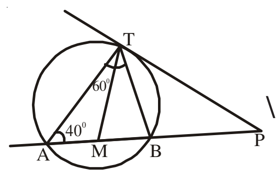
4. Two tangent segments PA and PB are drawn to a circle with centre O such that $\angle APB = 120^0$. Prove that $AP = \frac{1}{2} OP$.

5. In given figure O is centre of circle and $\angle PBQ = 40^0$, find



(i) $\angle QPY$ (ii) $\angle POQ$ (iii) $\angle OPQ$

6. In figure if $\angle PAT = 40^0$ and $\angle ATB = 60^0$, Show tha PM = PT.



STRETCH YOURSELF

1. With the help of an activity Show that a tangent is a line perpendicular to the radius through the point of contact.
2. A point O in the interior of a rectangle ABCD is joined to each of the vertices A, B, C and D, Prove that $OA^2 + OC^2 = OB^2 + OD^2$.

ANSWERS

CHECK YOUR PROGRESS :

5. (i) 40^0 (ii) 80^0 (iii) 50^0

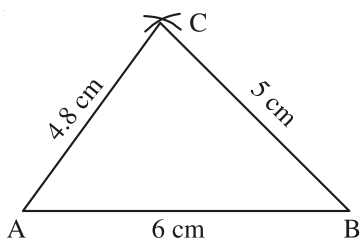
18

CONSTRUCTIONS

- **When 3 sides of a triangle are given :**

Steps:

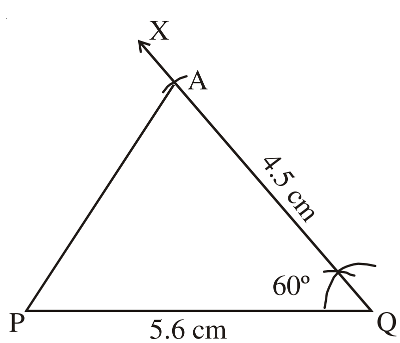
1. Draw AB = 6cm.
2. With A as centre and radius 4.8 cm draw an arc.
3. With B as centre and radius 5 cm draw another arc intersecting the previous arc at C.
4. Joint A to C and B to C. ΔABC is the required triangle.



- **When 2 sides and included angle are given :**

Steps:

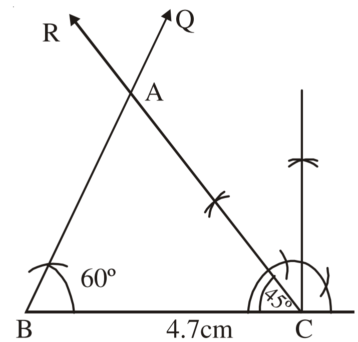
1. Draw PQ = 5.6 cm.
2. At Q construct an angle $\angle PQX = 60^0$.
3. With Q as centre and radius 4.5 cm draw an arc cutting QX at R.
4. Join P to R, ΔPQR is the required triangle.



- **When two angles and included side of Δ are given:**

Steps:

1. Draw BC = 4.7 cm.
2. At B construct ∠CBQ = 60º.
3. At C construct ∠BCR = 45º meeting BQ at A. ΔABC is the required triangle.

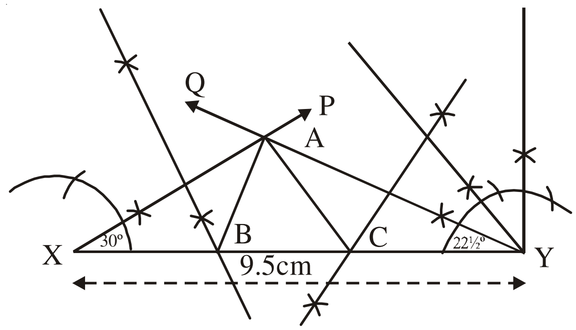


- **When perimeter and two base angles of a triangle are given :**

Steps:

1. Draw XY = 9.5cm
2. At X construct ∠YXP = 30º (Which is $\frac{1}{2}\times 60^0$).
3. At Y construct $\angle XYQ = 22\frac{1}{2}^0$ (which is $\frac{1}{2}\times 45^0$)
4. Draw right bisector of XA cutting XY at B.
5. Draw right bisector of YA cutting XY at C.
6. Join A to B and A to C.

ΔABC is the required triangle.



- **Construct a ΔABC when AB + AC = 8.2 cm, BC = 3.6 cm, ∠B = 45°**

Steps:

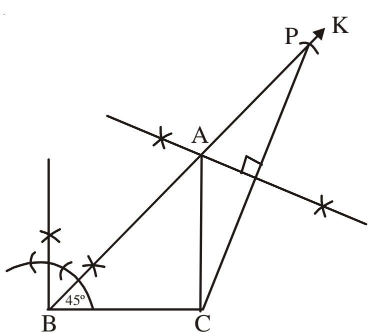
(1) Draw BC = 3.6 cm

(2) At B construct ∠CBK = 45°.

(3) From BK cutoff BP = 8.2 cm.

(4) Join C to P and draw right bisector of CP intersecting BP at A.

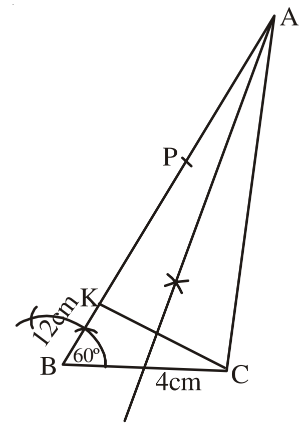
(5) Join A to C, ΔABC is the required triangle.



- **Construct a ΔABC, when BC = 4cm, ∠B = 60°, AB – AC = 1.2 cm**

Steps:

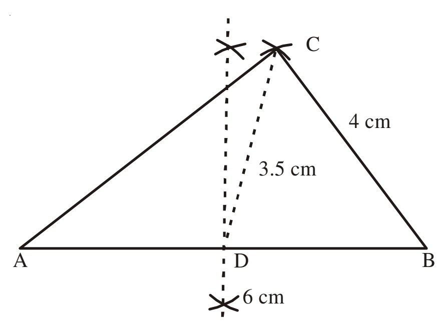
1. Draw BC = 4 cm.
2. Construct ∠CBP = 60°.
3. From BP cutoff BK = 1.2 cm.
4. Join C to K and draw right bisector of CK intersecting BP produced at A.
5. Join A to C, ΔABC is the required traingle.



- **Construct a ΔABC in which AB = 6cm, BC = 4cm and median CD = 3.5cm.**

Steps :

1. Draw AB = 6cm.
2. Draw right bisector of AB meeting AB in D.
3. With D as centre and radius 3.5cm draw an arc.
4. With B as centre and radius 4cm draw another arc intersecting the previous arc in C.
5. Join A to C and B to C, ΔABC as the required triangle.

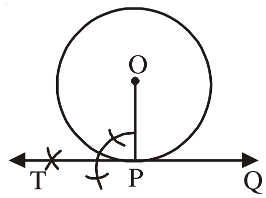


- **To draw a tangent to a given circle at a given point on it using its centre :**

Steps:

1. Draw a circle with centre O and a point P on it.
2. Joint O to P

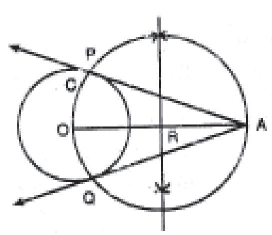
3. At P draw PT $\perp$ OP.
4. Produce TP to Q, then T PQ is the required tangent.



- **To draw tangents to a given circle from a given point outside it**

Steps:

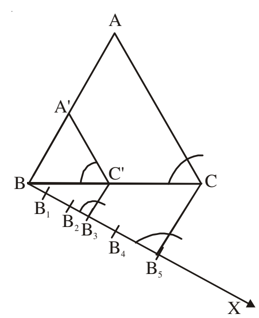
1. Draw a circle with centre O and a point P outside it.
2. Join O to A.
3. Draw the right bisector of OA. Let R be the mid point of OA.
4. With R as centre and radius as RO. Draw a circle intersecting the given circle at P and Q.
5. Join A to P and A to Q, then AP and AQ are required tangents.



- **To construct a triangle similar to a given triangle with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle.**

Steps:

1. Let ABC be the given Δ. Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
2. Locate 5 points B_1, B_2, B_3, B_4 and B_5 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.
3. Join B_5 to C and draw a line through B_3 parallel to B_5C to meet BC at C′.
4. Draw a line through C parallel to CA to meet AB in A′. The $\Delta A'BC'$ is the required triagle.



CHECK YOUR PROGRESS:

1. Using a ruler and compass it is possible to construct an angle of:
   (A) 37.5^0 (B) 25^0 (C) 40^0 (D) 70^0.
2. The constuction of a ΔPQR in which PQ = 5cm, $\angle A = 60^0$ is not possible, when difference of QR and PR is equal to:
   (A) 5.2cm (B) 4.8 cm (C) 3.7cm (D) 4.5cm
3. The construction of ΔPQR is not possible, in which PQ = 5.5cm $\angle Q = 45^0$, and PQ + RP is:
   (A) 5cm (B) 6cm (C) 7cm (D) 8cm

4. The construction of a ΔABC given that $BC = 3cm$, $\angle C = 60^0$ is possible when difference of AB and AC is equal to:

   (A) 4cm (B) 3.5cm (C) 3.1cm (D) 2.4cm

5. Draw a line segment $BA = 8cm$, find point C on it such that $AC = \frac{3}{4}AB$.
6. Construct a triangle PQR, given that $PQ = 3.4cm$, $QR = 5.2cm$ and $PR = 7.5cm$.
7. Construct a triangle ABC, given that $AC = 5.5cm$, $AB = 3.2cm$ and $\angle A = 135^0$.
8. Construct a triangle PQR given that $QR = 3.2cm$, $\angle Q = 85^0$ and $\angle R = 60^0$.
9. Construct a triangle ABC in which $\angle B = 60^0$, $\angle C = 45^0$ and $AB + BC + CA = 11cm$.

STRETCH YOURSELF

1. Construct a triangle PQR in which $QR = 8cm$, $\angle Q = 45^0$ and $PQ – PR = 3.5cm$.
2. Construct a ΔABC in which $BC = 5$ cm, $\angle B = 60^0$ and $AB + AC = 7.5cm$.
3. Construct a triangle ABC in which $AB = 5$ cm, $BC = 4.2$ cm and median $CD = 3.8$ cm.
4. Draw triangle PQR having base $QR = 6cm$, $\angle PQR = 60^0$ and side $PQ = 4.5$ cm. Construct a triangle $P'QR'$ simialr to ΔPQR with scale factor $\frac{4}{5}$.

ANSWERS

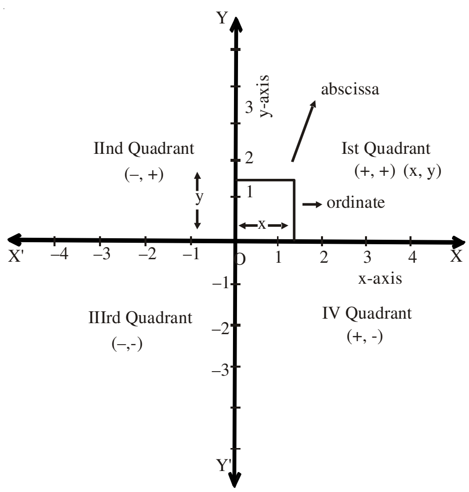
CHECK YOUR PROGRESS :

1. A
2. A
3. A
4. D

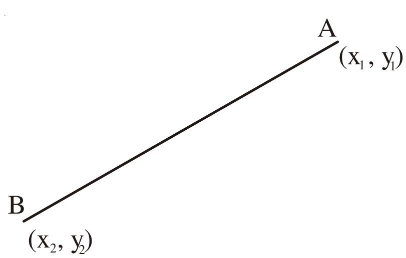
19

CO-ORDINATE GEOMETRY

- Any point (x, 0) lies on x-axis.
- Any point (0, y) lies on y-axis.
- (x, y) and (y, x) do not represent the same point when $x \neq y$.
- Co- ordinates of origin are (0, 0) .



- Distance between two points A (x_1, y_1) and B (x_2, y_2), $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



- Three points A, B and C are collinear, if AB + BC = AC
- A quadrilateral will be a :

Parallelogram: If length of oposite sides are equal.

Rectangle: If opposite sides are equal and diagonals are equal.

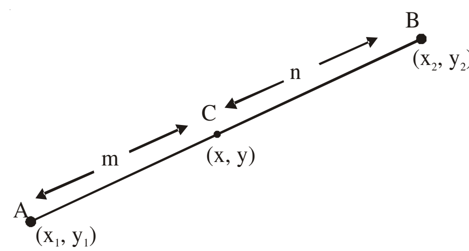
Square: If all 4 sides are equal, diagonals are also equal.

Rhombus: If all 4 sides are equal

Parallelogram but Not rectangle: Opposite sides are equal but diagonals are not equal

Rhombus but not square: All sides are equal but diagonals are not equal.

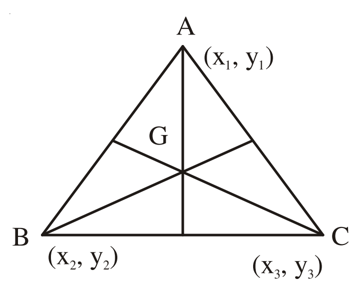
- **Section formula:**



$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$\text{Mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- **Centroid:**



$$G(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

CHECK YOUR PROGRESS:

1. A triangle has vertices (0,8), (0,0) and (6,0). Its perimeter is :

   (A) 10 (B) 24 (C) 12 (D) 14

2. The point which divides the line segment joining the points (–8, –5) and (–2, –10) in the ratio 2:1 internally lies in the :

   (A) Ist quadrant (B) IInd quadrant (C) IIIrd quadrant (D) IVth quadrant

3. If $\left(\frac{a-2}{2}, 5\right)$ is the mid point of the line segment joining the points (1,7) and (–5, 3), the value of a is :

   (A) 2 (B) 0 (C) –4 (D) –3

4. The distance between (6, x) and (0, 4) is 10. The value of x is :

   (A) 4 or 12 (B) 4 or –12 (C) –4 or 12 (D) –4 or –12

5. A point on x-axis which is equidistant from A (5,4) and B (–2, 3) is :

   (A) (–1, 0) (B) (1, 0) (C) (2, 0) (D) (–2, 0)

6. Plot the points (–3, –2), (–1, –2), (–2, 0), (–3, –1) and join them in the order. What figure you get?

7. The length of a line segement is 10 units. If one end is at (2, –3) and abscissa of the other is 10, show that its ordinate is either 3 or –9.

8. If A and B are (1, 4) and (5, 2) respectively, find co-ordinates of the point P on AB so that 4 AP = 3 PB.

9. Show that the points A (3, 3), B (–1, 0) and C (1, 4) form a right triangle whose hypotenuse is AB.

10. Show that the points P (0, –4), Q (6, 2), R (3, 5) and S (–3, –1) are the vertices of the rectangle PQRS.

STRETCH YOURSELF

1. AB is a line segment with co-ordinates as A (9, 2) and B (–5, 12). In what ratio point (3, 2) divides the line segment AB.

2. Find the co-ordinates of the points which divide the line segment joining the points (–4, 0) and (0, 6) in four equal parts.

3. Points A(–5, 0), B(0, 15) and C(–10, 20) are vertices of a triangle ABC. Point P lies on side AB and divides it in the ratio 2 : 3. Similarly point Q lies on the side AC and divides it in the ratio 2 : 3

   (i) Find the co-ordinates of the points P and Q.

   (ii) Show that PQ = $\frac{2}{5}$ BC.

ANSWERS

CHECK YOUR PROGRESS :

1. B 2. B 3. C 4. C

5. C 6. Pentagon 8. $\left(\frac{19}{7}, \frac{22}{7}\right)$

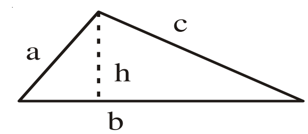
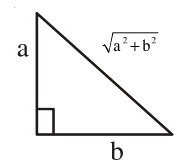
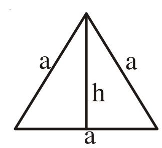
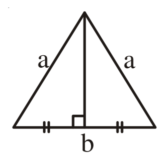
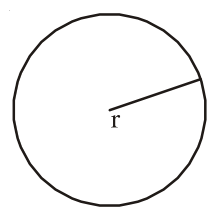
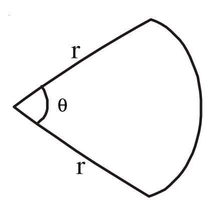
STRETCH YOURSELF :

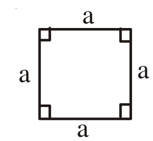
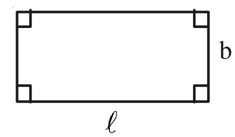
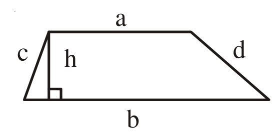
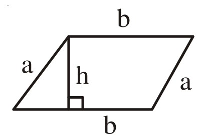
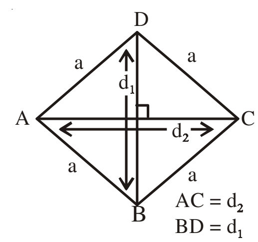
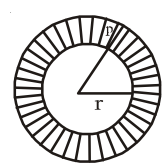
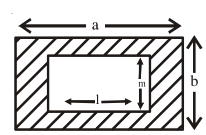
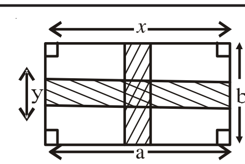
1. 3 : 4 2. $\left(-3, \frac{3}{2}\right)$, $(-2, 3)$, $\left(-1, \frac{9}{2}\right)$

3. $\left(-5, \frac{45}{2}\right)$, $(-20, 30)$

20

PERIMETERS AND AREAS OF PLANE FIGURES

Name of the Figure	**Perimeter/ circumference**	**Area**	**Figure**
Triangle	$a+b+c$	$\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\dfrac{a+b+c}{2}$ or $\dfrac{1}{2}bh$	
Right angled triangle	$a+b+\sqrt{a^2+b^2}$	$\dfrac{1}{2}ab$	
Equilateral triangle	$3a$	$\dfrac{\sqrt{3}}{4}a^2$	
Isosceles triangle	$2a+b$	$\dfrac{b}{4}\sqrt{4a^2-b^2}$	
Circle	$2\pi r$	πr^2	
Sector of a circle	$\dfrac{\pi r\theta}{180}+2r$ (θ is in degrees)	$\dfrac{\theta}{360}\times\pi r^2$	

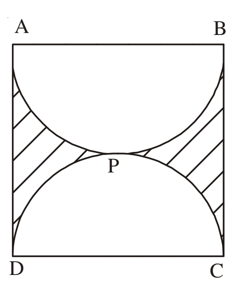
Square	$4a$	a^2	
Rectangle	$2(\ell + b)$	$\ell \times b$	
Trapezium	$a + b + c + d$	$\frac{1}{2}(a + b)h$	
Parallelogram	$2(a + b)$	bh	
Rhombus	$4a$	$\frac{1}{2} \times d_1 \times d_2$	
Circular Path	$2\pi(R + r)$	$\pi R^2 - \pi r^2$	
Rectangular Path		$ab - lm$	
		$ay + bx - xy$	

CHECK YOUR PROGRESS:

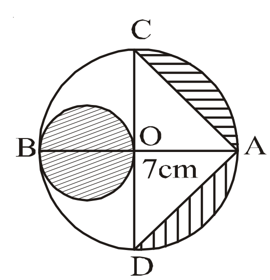
1. The area of a rectangular field is 3630 sq. m and its sides are in the ratio 6:5. The perimeter of the field is :
   (A) 363m (B) 121m (C) 242m (D) 484m
2. The area of a plot in the shape of a quadrilateral, one of whose diagonals is of length 30 m and the lengths of perpendiculars from the opposite vertices are 10 m and 16 m respectively is :
   (A) $480m^2$ (B) $780m^2$ (C) $160m^2$ (D) $300m^2$
3. The difference between the parallel sides of a trapezium of area 390 cm^2 is 12 cm. If the distance between the parallel sides is 15cm then lengths of two parallel sides in cm are :
   (A) 26, 14 (B) 27, 15 (C) 36, 24 (D) 32, 20
4. The difference in the circumference and diameter of a circle is 15cm. The radius of the circle is [use $\pi=\frac{22}{7}$] :
   (A) 7cm (B) $\frac{7}{2}$ cm (C) 3cm (D) $\frac{9}{2}$ cm
5. From a cicular cardboard of radius 10.5 cm, a sector of central angle 60^0 is cut out. The area of the remaining part of the cardboard is [use $\pi=\frac{22}{7}$] :
   (A) $228\frac{2}{3}cm^2$ (B) $128\frac{2}{3}cm^2$ (C) $228\frac{1}{3}cm^2$ (D) $128\frac{1}{3}cm^2$
6. Two perpendicular paths of width 5m each run in the middle of a rectangular park of dimension 100 m $\times$ 60m, one parallel to the lenth and the other parallel to the width. Find the cost of constructing these paths at the rate of Rs. 6 per m^2. Also find the cost of cultivating the remaining part at the rate of Rs. 3 per m^2.
7. The side of a rhombus is 10 cm and one of its diagonals is of length 12cm. Find the length of the other diagonal of the rhombus and its area. Also, find the breadth of a rectangle of length 12 cm whose area is equal to area of the rhombus.

STRETCH YOURSELF

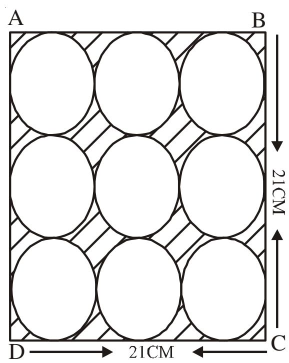
1. In a square ABCD of side 21cm, two semicircles APB and DPC have been drawn. Find the area of i) Unshaded region. ii) Shaded region [use $\pi=\frac{22}{7}$]



2. In the figure AB is diameter of a circle of radius 7cm. If CD is another diameter of the circle. Find the area of the shaded region. [use $\pi=\frac{22}{7}$]



3. ABCD is a square of side 21 cm. Nine congruent circles, each of radius 3.5cm are inscribed in the square, touching all the sides of the square. find the areas of the i) Unshaded region ii) Shaded region.



ANSWERS

CHECK YOUR PROGRESS :

1. C 2. B 3. D 4. B

5. A 6. Rs. 4650, Rs. 15675

7. d_2 =16cm, Area = 96 cm^2, Breadth of rectangle = 8cm.

STRETCH YOURSELF :

1. (i) 346.5 cm^2 (ii) 94.5cm^2

2. $\left(\frac{235}{32}\right)cm^2$

3. (i) 346.5 cm^2 (ii) 94.5 cm^2

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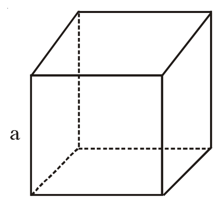
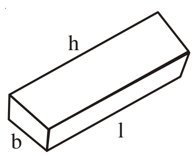
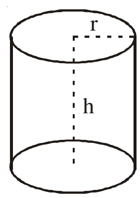
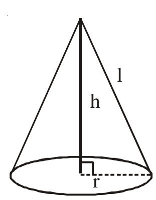
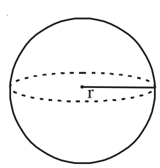
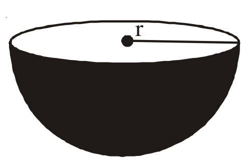
SURFACE AREAS AND VOLUMES OF SOLID FIGURES

Name of the solid	**Figure**	**Lateral Surace Area**	**Total surface Area**	**Volume**
Cube		$4a^2$	$6a^2$	a^3
Cuboid		$2h(l+b)$	$2(lb+bh+lh)$	lbh
Cylinder		$2\pi rh$	$2\pi r(r+h)$	πr^2h
Cone		πrl	$\pi r(l+r)$	$\frac{1}{3}\pi r^2h$
Sphere	- 	$4\pi r^2$		$\frac{4}{3}\pi r^3$
Hemisphere		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$

CHECK YOUR PROGRESS:

1. The radius and height of a right circular cylinder are $10\frac{1}{2}$ cm and 12cm respectively. Its curved surface area is (use $\pi = \frac{22}{7}$) :

   (A) 396cm^2 (B) 792cm^2 (C) 1188cm^2 (D) 132cm^2

2. The volume of a right circular cylinder is 4620 cm^3 and its base radius is 14cm. The curved surface area of the cylinder is (use $\pi = \frac{22}{7}$) :

   (A) 330cm^2 (B) 440cm^2 (C) 660cm^2 (D) 990cm^2

3. The base radius and height of a right circular cone are 3.5cm and 12cm respectively. Its curved surface area is (use $\pi = \frac{22}{7}$) :

   (A) 550cm^2 (B) 137.5cm^2 (C) 275cm^2 (D) 12.5cm^2

4. The volume of a hemispherical bowl is 2425.5 cm^3. The radius of the hemisphere is :

   (A) 5.25cm (B) 10.5cm (C) 15.75cm (D) 12cm

5. The surface area of a sphere is 1386cm^2. Its volume is :

   (A) 9702cm^3 (B) 2425.5cm^3 (C) 441cm^3 (D) 4851cm^3

6. If the surface area of a cube is 864 cm^2, find its side and volume.

7. The radius of a road roller is 42cm and it is 1 meter long. If it takes 250 revolutions to level a playground, find the cost of levelling the ground at the rate of Rs. 5 per sq. m (use $\pi = \frac{22}{7}$)

8. A conical tent is 3m high and its base radius is 4m. Find the cost of canvas required to make the tent at the rate of Rs. 50 per m^2 (use $\pi = 3.14$)

9. The diameter of a solid hemispherical toy is 35 cm, find its

   (i) Curved surface area

   (ii) Total surface area

   (iii) Volume

10. The base radii of two right circular cylinders of the same height are in the ratio 3 : 5. Find the ratio of their volumes.

STRETCH YOURSELF

1.. The radius and height of a closed right circular cylinder are in the ratio 5:7 and its volume is 4400cm^3. Find the radius and height of the cylinder

[use $\pi = \frac{22}{7}$]

2.. A metallic solid ball of diameter 28cm is melted to form solid cylinders of base radius 7cm and height $9\frac{1}{3}$ cm. Find the number of cylinders so formed.

3. The radii of two cylinders are in the ratio 7:6 and their heights are in the ratio 3 : 4. Find the ratio of their

(i) Volumes

(ii) Curved surface areas.

ANSWERS

CHECK YOUR PROGRESS :

1. B 2. C 3. B 4. B

5. D

6. Side = 12cm, Volume = 1728 cm^3

7. Rs. 3300/- 8. Rs. 3140/-

9. Curved surface area = 1925 sq cm, Total surfacearea= 2887.5 sq cm, Volume = 11229.17cm^3,

10. 9 : 25

STRETCH YOURSELF :

1. Radius = 10 cm, height = 14 cm

2. 8

3. (i) 49 : 48 (ii) 7 : 8

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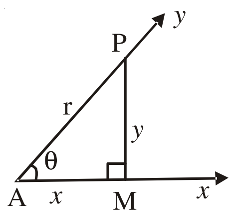
INTRODUCTION TO TRIGONOMETRY

- **Trigonometry** : Trigonometry is that branch of mathematics which deals with the measurement of the sides and the angles of a triangle and the problems related to angles.



- **Trigonometric Ratios :** Ratios of the sides of a triangle with respect to its acute angles are called trigonometric ratios.
 In the right angled Δ AMP
 For acute angle PAM = θ
 Base = AM = x, Perpendicular = PM = y, Hypotenuse = AP = r

 Here, sine $\theta = \frac{y}{r}$, Written as sin θ

 cosine $\theta = \frac{x}{r}$, Written as cos θ

 tangent $\theta = {}^{y}\!/_{x}$, Written as tan θ

 cosecent $\theta = {}^{r}\!/_{y}$, Written as cosec θ

 secent $\theta = {}^{r}\!/_{x}$, Written as sec θ

 cotangent $\theta = {}^{x}\!/_{y}$, Written as cot θ

 $\Rightarrow$ $\sin\theta$, $\cos\theta$, $\tan\theta$ etc. are complete symbols and can not be separated from θ.
 $\Rightarrow$ Every trigonometric ratio is a real number.
 $\Rightarrow$ θ is restricted to be an acute angle.
 $\Rightarrow$ For convenience, we write $(\sin\theta)^2$, $(\cos\theta)^2$, $(\tan\theta)^2$ as $\sin^2\theta$, $\cos^2\theta$ and $\tan^2\theta$ respectively.

- **Relation between Trigonometric ratios :**

 $\Rightarrow \sin\theta = \frac{1}{\operatorname{cosec}\theta}$ or $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ or $\sin\theta \times \operatorname{cosec}\theta = 1$

 $\Rightarrow \cos\theta = \frac{1}{\sec\theta}$ or $\sec\theta = \frac{1}{\cos\theta}$ or $\cos\theta \times \sec\theta = 1$

 $\Rightarrow \tan\theta = \frac{1}{\cot\theta}$ or $\cot\theta = \frac{1}{\tan\theta}$ or $\tan\theta \times \cot\theta = 1$

 $\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$

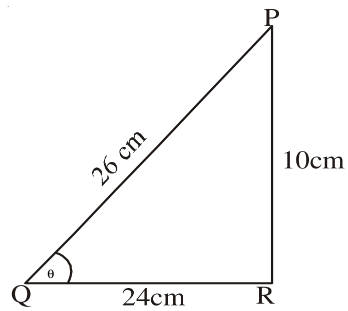
- **Trigonometric Identities :** An equation involving trigonometric ratios of an angle θ is said to be a trigonometric identity if it is satisfied for all values of θ for which the given trigonometric ratios are defined.
 Some special trigonometric Identities
 $\Rightarrow \sin^2\theta + \cos^2\theta = 1$ or $1 - \cos^2\theta = \sin^2\theta$ or $1 - \sin^2\theta = \cos^2\theta$.
 $\Rightarrow 1 + \tan^2\theta = \sec^2\theta$ or $\sec^2\theta - \tan^2\theta = 1$ or $\sec^2\theta - 1 = \tan^2\theta$
 $\Rightarrow 1 + \cot^2\theta = \operatorname{cosec}^2\theta$ or $\operatorname{cosec}^2\theta - \cot^2\theta = 1$ or $\operatorname{cosec}^2\theta - 1 = \cot^2\theta$.

- **Trigonometric ratios of complementary angles:** If θ is an acute angle then
 $\sin(90° - \theta) = \cos\theta$ and $\cos(90° - \theta) = \sin\theta$
 $\tan(90° - \theta) = \cot\theta$ and $\cot(90° - \theta) = \tan\theta$
 $\operatorname{cosec}(90° - \theta) = \sec\theta$ and $\sec(90° - \theta) = \operatorname{cosec}\theta$
 Here θ is an acute angle and $(90° - \theta)$ is a complementary angle for θ.

- **Finding of trigonometric ratios :** $\Rightarrow$ If two sides of any right triangle are given, then all the six trigonometric ratios can be written.
 $\Rightarrow$ If one trigonometric ratio is given, then other trigonometic ratios can be written by using pythagoras theorem or trigonometic identities.

CHECK YOUR PROGRESS:

1. In the given figure, which of the following is correct?



(A) $\sin\theta + \cos\theta = \frac{17}{13}$ (B) $\sin\theta - \cos\theta = \frac{17}{13}$

(C) $\sin\theta + \sec\theta = \frac{17}{13}$ (D) $\tan\theta + \sec\theta = \frac{17}{13}$

2. If $5\tan\theta - 4 = 0$, the value of $\frac{5\sin\theta - 4\cos\theta}{5\sin\theta + 4\cos\theta}$ is :

(A) $\frac{5}{3}$ (B) $\frac{5}{6}$ (C) 0 (D) $\frac{1}{6}$

3. The value of $\left(\frac{\sin\theta \,.\, \cos(90^0 - \theta)}{\sin(90^0 - \theta).\cos\theta} + 1\right)$ is equal to :

(A) $\sin\theta + \cos\theta$ (B) $\cos^2\theta$ (C) $\sec^2\theta$ (D) $\text{cosec}^2\theta$

4. The value of $\frac{\sec 41^0 \operatorname{cosec} 49^0 - \tan 41^0 \cot 49^0}{\sec 41^0 . \sin 49^0 + \cos 49^\circ \,.\operatorname{cosec} 41^0}$ is :

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) 0

5. If $\sin(\theta + 36^0) = \cos\theta$ and $\theta + 36^0$ is an acute angle, then θ is equal to :

(A) 54^0 (B) 18^0 (C) 21^0 (D) 27^0

6. If $\cot\theta = \frac{12}{5}$, find the value of $\frac{\sin\theta.\cos\theta}{\sec\theta}$.

7. Prove that $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = 2\operatorname{cosec} A$.

8. If $\cos\theta = \frac{1}{2}$ and $\text{Sin}\,\theta = \frac{\sqrt{3}}{2}$, find the value of $\sec\theta$, $\text{cosec}\,\theta$ and $\tan\theta$.

STRETCH YOURSELF

1. For a right angled Δ ABC, right angled at C, tanA=1, find the value of $\sin^2B.\cos^2B$.
2. Find the value of $\tan1^0. \tan2^0.\tan3^0.........\tan89^0$.

ANSWERS

CHECK YOUR PROGRESS :

1. A 2. C 3. C

4. C 5. D 6. $\frac{720}{2197}$

8. $\sec\theta = 2$, $\text{cosec}\,\theta = \frac{2}{\sqrt{3}}$, $\tan\theta = \sqrt{3}$

STRETCH YOURSELF :

1. $\frac{1}{4}$ 2. 1

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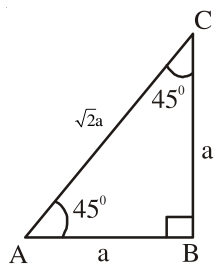
TRIGONOMETRIC RATIOS OF SOME SPECIAL ANGLES

- **Trigonometic ratios of angle 45^0 :** In Δ ABC, $\angle B = 90^0$, $\angle A = 45^0$ then $\angle C = 45^0$ and AB = BC = a then AC = $\sqrt{2}a$

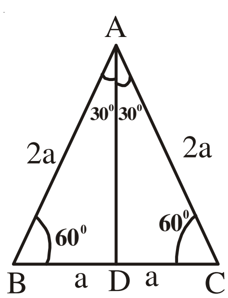


In Δ ABC, $\sin 45^0 = \frac{1}{\sqrt{2}}$, $\cos 45^0 = \frac{1}{\sqrt{2}}$,

$\tan 45^0 = 1$, $\cot 45^0 = 1$, $\text{cosec}\, 45^0 = \sqrt{2}$,

$\sec 45^0 = \sqrt{2}$

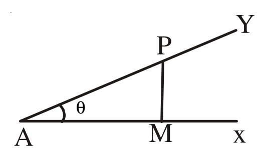
- **Trigonometric ratios of 30^0 and 60^0 :** In an equilateral triangle ABC with side 2a, AD = $\sqrt{3}a$



In Δ ADB,	In Δ ADB
$\text{Sin } 30^0 = \frac{1}{2}$,	$\sin 60^0 = \frac{\sqrt{3}}{2}$
$\cos 30^0 = \frac{\sqrt{3}}{2}$	$\cos 60^0 = \frac{1}{2}$

$\tan 30^0 = \frac{1}{\sqrt{3}}$	$\tan 60^0 = \sqrt{3}$
$\text{cosec}\, 30^0 = 2$	$\text{cosec}\, 60^0 = \frac{2}{\sqrt{3}}$
$\sec 30^0 = \frac{2}{\sqrt{3}}$	$\sec 60^0 = 2$
$\cot 30^0 = \sqrt{3}$	$\cot 60^0 = \frac{1}{\sqrt{3}}$

- **Ttrigonometric Ratios of 0^0 and 90^0:** Let $\angle XAY = \theta$.



In Δ AMP, we have

$\sin\theta = \frac{PM}{AP}$, $\cos\theta = \frac{AM}{AP}$, $\tan\theta = \frac{PM}{AM}$

If θ becomes 0^0, then PM = 0 , AM = AP

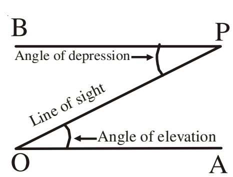
If θ becomes 90^0, then AM = 0, AP = PM

If $\theta = 0^0$, then	If $\theta = 90^0$, then
$\sin 0^0 = 0$	$\sin 90^0 = 1$
$\cos 0^0 = 1$	$\cos 90^0 = 0$
$\tan 0^0 = 0$	$\tan 90^0 =$ Not defined
$\text{cosec}\, 0^0 = \frac{1}{0} =$ not defined	$\text{cosec}\, 90^0 = 1$
$\sec 0^0 = \frac{1}{1} = 1$	$\sec 90^0 =$ not defined
$\cot 0^0 = \frac{1}{0} =$ not defined	$\cot 90^0 = 0$

Trigonometric Ratios of $0^0, 30^0, 45^0, 60^0$ and 90^0

ratio \ $\theta \rightarrow$	0^0	30^0	45^0	60^0	90^0
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
cot	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
cosec	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined

- **Applications of trigonometry :**



- **Line of sight:** If an observer is at O and the point P is under consideration then the line OP is called line of sight of the point P.
- **Angle of elevation:** Angle between the line of sight and the horizontal line OA is known as angle of elevation of point P as seen from O.
- **Angle of depression:** If an observer is at P and the object under consideration is at O , then the $\angle BPO$ is known as angle of depression of O as seen from P.
- **Relation between angle of elevation and angle of depression**: Angle of elevation of a point P as seen form O is equal to the angle of depression of O as seen from P.

CHECK YOUR PROGRESS:

1. $\dfrac{2\tan 30^0}{1+\tan^2 30^0} + \dfrac{1-\tan 45^0}{1+\tan^2 45^0}$ is equal to :

   (A) $\sin 60^0$ (B) $\sin 30^0$ (C) $\cos 60^0$ (D) $\tan 60^0$

2. The ratio of the length of a rod to its shadow is $1 : \sqrt{3}$. The angle of elevation of sun is:

   (A) 30^0 (B) 45^0 (C) 60^0 (D) 90^0

3. Evaluate $\tan^2 45^0 - \sin^2 60^0 + 2\cos^2 30^0$.

4. In Δ ABC, right angled at C, AC = $2\sqrt{3}$ cm and BC = 2cm. Find $\angle$A and $\angle$B.

5. If $\sin(A+B) = 1$ and $\cos(A-B) = \dfrac{\sqrt{3}}{2}$, $0^0 < A+B \leq 90^0$ and $A \geq B$ find A and B.

STRETCH YOURSELF

1. The string of a kite is 100m long and it makes an angle of 60^0 with the horizontal. Assuming that there is no slack in the string, calculate the height of the kite.
2. A 12m heigh tree is broken by the wind in such a way that its top touches the ground and makes an angle of 30^0 with the ground. Find the height at which tree is broken.
3. Find the value of A, if $\sin 2A = 2\sin A$ where $0 \leq A < 90^0$.

ANSWERS

CHECK YOUR PROGRESS :

1. A
2. A
3. $\dfrac{7}{4}$
4. $\angle A = 30^0$, $\angle B = 60^0$
5. $\angle A = 60^0$, $\angle B = 30^0$

STRETCH YOURSELF :

1. $50\sqrt{3}$ m
2. 4m
3. 0^0

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DATA AND THEIR REPRESENTATION

- **Statistics:** Statistics is a branch of mathematics which deals with collection, presentation, analysis, interpretation of data and drawing of inferences/conclusions there from.
- **Data:** Facts or figures, which are numerical or otherwise collected with a definite purpose.
- **Types of Data:**

  Primary Data: Data which an investigator collects for the first time for his own purpose.

  Secondary Data: Data which the investigator obtains from some other source, agency or office for his own purpose
- **Presentation of Data:**

  Raw or Ungrouped Data: The data obtained in original form and presented ungrouped without any re-arrangement or condensed form

  An Array: The Presentation of a data in ascending or descending order of magnitude.

  Grouped Data: Rearrangement or condensed form of data into classes or groups.
- **Range of Data:** Difference between the highest and lowest values in the data.
- **Frequency:** The number of times an observation occurs in data.
- **Class Interval:** Each group in which the observations/values of a data are condensed.
- **Class limits:** Values by which each class interval is bounded . Value on the left is called lower limit and value on the right is called upper limit.
- **Class size:** Difference between the upper limit and the lower limit.
- **Class mark of a class interval:** Mid value of a class interval $= \dfrac{\text{lower limit} + \text{upper limt}}{2}$
- **Cumulative Frequency of a class:** Total of frequencies of a particular class and of all classes prior to that class.
- **Graphical Representation of Data:**

  Bar Graph: A pictorial representation of data in which usually bars of uniform width are drawn with equal spacing between them on one axis and values of variable (frequencies) are shown on other axis.

  Histogram: A pictorial representation like bar graph with no space between the bars. It is used for continuous grouped frequency distribution.

  Frequency Polygon: A graphical representation of grouped frequency distribution in which the values of the frequencies are marked against the class mark of the intervals and the points are joined by line segments.

CHECK YOUR PROGRESS:

1. The class mark of the class 90 -120 is .

   (A) 90 (B) 105 (C) 115 (D) 120

2. In a given data some variables are given with particular values, we want to represent these graphically, then we can represent these, using-

   (A) Histogram (B) Frequency Polygon

   (C) Bargraph (D) None

3. The range of the data 25, 18, 20, 22, 16, 6, 17, 15, 12, 30, 32, 10, 19, 8, 11, 20 is -

   (A) 10 (B) 15 (C) 18 (D) 26

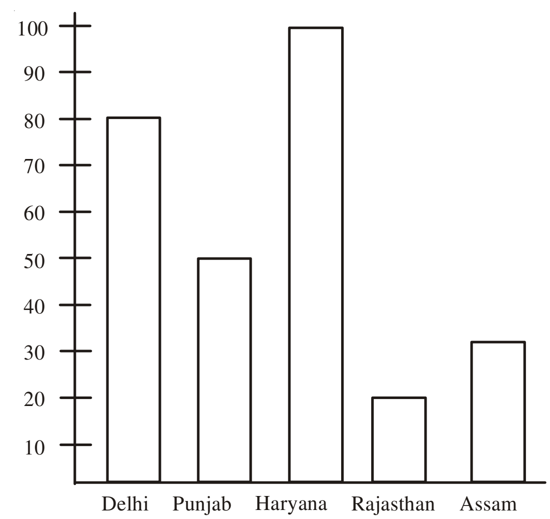
4. When the information is gathered from a source which already has the information stored, the data obtained is called-
   (A) Primary data (B) Secondary Data
   (C) Frequency data (D) Raw data
5. Class size of the class interval 5 -15 is -
   (A) 5 (B) 1 (C) 10 (D) 20
6. Upper class limits of the class interval 30 - 39 is-
   (A) 30 (B) 34 (C) 39 (D) 35
7. In a frequency distribution the mid value of a class is 10 and the width of the class is 6. The lower limit of the class is -
   (A) 6 (B) 7 (C) 8 (D) 12
8. Facts or figures collected with a definite purpose are called-
   (A) data (B) Histogram (C) Median (D) mode

STRETCH YOURSELF

1. The following cumulative frequency distribution table shows the marks obtained by 55 students of classX. Represent this as frequency distribution table

Marks	Cumulative Frequency
Less than 5	2
Less than 10	8
Less than 15	21
Less than 20	38
Less than 25	49
Less than 30	53
Less than 35	55

2. The bar graph given below represents the tickets of different state lotteries sold by an agent on a day. Read the bar graph and answer the following:
   (i) How many tickets of Assam state lottery were sold.
   (ii) Of which state, maximum tickets were sold?
   (iii) Of which state, minimum tickets were sold?
   (iv) Say true or false: The Maximum number of tickets is three times the minimum number of tickets.



ANSWERS

CHECK YOUR PROGRESS :

1. B	2.C	3. D	4. B
5. C	6. C	7. B	8. A

STRETCH YOURSELF :

1.

Marks	No. of Students
0 - 5	2
5 - 10	6
10 - 15	13
15 - 20	17
20 - 25	11
25 - 30	4
30 - 35	2

2. (i) 30 (ii) Haryana
(iii) Rajasthan
(iv) False

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MEASURES OF CENTRAL TENDENCY

- **Central Tendency:** A Single quantity which enables us to know the average characteristics of the data under consideration. Use of central tendency is a technique to analyse the data.
- **Various Measures of central tendency :**
  The arithmetic mean/the mean/Average
  The median
  The mode
  Mean: It is the ratio of the sum of all values of the variable and the number of osbervations and denoted by $\overline{X}$.

  Mean for raw data $\quad \overline{x} = \dfrac{\sum_{t=i}^{n} \times i}{n}$

  $x_1, x_2, x_3....X_n$ are n observations, symbol $\sum$ stand for 'sum of'
  Mean for ungrouped frequency distribution:

  $$\overline{x} = \frac{\sum_{i=1}^{n} fi \times i}{\sum_{o=1}^{n} fi}$$

  $x_1, x_2, x_3, ... x_n$ are n observations with their respective frequencies $f_1, f_2, f_3, ...f_n$.
  Mean for grouped frequency distribution:

  $$\overline{x} = \frac{\sum_{i=1}^{n} fi \times i}{\sum_{i=1}^{n} fi}$$

  $x_1, x_2, x_3, ... x_n$ are class marks of different classes with their respective frequencies $f_1, f_2, f_3, ...f_n$.

  Mean by short cut Method:

  $$\overline{X} = A + \frac{\sum_{i=1}^{n} fidi}{\sum_{i=1}^{n} fi} \times C$$

  A = assumed mean

  $$di = \frac{x_i - A}{C}$$

  C = class size
- **Median :** It is the middle most value of arrayed data. It divides the arrayed data into two equal parts.

  Median, when number of observations is odd:

  Median = value of $\left(\frac{n+1}{2}\right)^{th}$ observation in the array, n is number of observations in arrayed form Median, when number of observations is even:

  $$\text{Median} = \frac{\text{value of}\left(\frac{n}{2}\right)^{th}\text{observation} + \text{value of}\left(\frac{n}{2}+1\right)^{th}\text{observation}}{2}$$
- **Mode:** It is the most frequenty occuring value amongst the given values of the variate in the data and denoted by Mo.

  or

  It is an observation with the maximum frequency in the given data.

CHECK YOUR PROGRESS:

1. The mean of the distribution in which variates are 1, 2, 3 .....n and the frequency of each being 1, is :

   (A) $\dfrac{n(n+1)}{2}$ (B) $\dfrac{n}{2}$ (C) $\dfrac{(n+1)}{2}$ (D) $n(n+1)$

2. Which of the following cannot be determined graphically?:

   (A) Mean (B) Median (C) Mode (D) None of these

3. The mean of 7 observations is 15. If each observation is increased by 2, then the new mean is :

(A) 15 5(B) 9 (C) 17 (D) 7

4. If the mean of the follwoing distribution is 2.6, then the value of y is:

Variable(x) :	1	2	3	4	5
Frequency:	4	5	y	1	2

(A) 3 (B) 8 (C) 13 (D) 24

5. The median of the first 10 prime numbers is:

(A) 11 (B) 12 (C) 13 (D)14

6. If the mean of 6, 7, x, 8, y, 14, is 9, then

(A) x + y = 21 (B) x + y = 19 (C) x - y = 19 (D) x - y = 21

7. What is the mode of 2, 7, 6, 7, 21, 5, 5, 10, 13, 7?

8. What is the median of 4, 8, 9, 11, 13, 17, 18, 19?

9. Find the mean of the following :

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	5	18	15	16	6

10. A group of 10 items has arithmetic mean 6. If the arithmetic mean of 4 of these items is 7.5, then find the mean of the remaining items.

STRETCH YOURSELF

1. If the mean of the follwoing frequency distribution is 62.8 and sum of all the frequencies is 50, then find the missing frequencies F_1 and F_2.

Class	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120
Frequency	5	F_1	10	F_2	7	8

2. The mean of n observations is $\overline{x}$. If the first item is increased by 1, second by 2 and so on, then find the new mean.

ANSWERS

CHECK YOUR PROGRESS :

1. C 2. A 3. C 4. B 5. B 6. B 7. 7 8. 12
9. 27 10. 15

STRETCH YOURSELF :

1. $F_1 = 8$, $F_2 = 12$ 2. $\overline{x} + \frac{n+1}{2}$

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PROBABILITY

- **Probability:** Probability is that branch of mathematics which deals with the measure of uncertainity in various phenomenon that gives several results/out comes instead of a particular one.
- **Definition of probability:** Numerical measure of 'Uncertainity' and denoted by P(E).
- **Experiment:** An activity which produce some well defined outcomes
- **Random Experiment**: An experiment in which all possible outcomes are known but the results can not be predicted in advance.
- **Trial:** Performing an experiment.
- **Outcome**: Result of the trial
- **Equally likely outcomes:** Outcomes which have equal chances of occurance.
- **Sample space** : Collection of all possible outcomes
- **Some special sample spaces:**

Coin tossed once $n(s) = 2 = 2^1$	**S = {H, T},**
Coin tossed twice or two coins tossed simultaneously	S = {HH, HT, TH, TT}, $n(s) = 4 = 2^2$.
Coin tossed thrice or three coins tossed simultaneously	S = {HHH, HTH, HHT, THH, TTT, TTH, THT, HTT}, $n(s) = 8 = 2^3$.
Die is thrown once	S = {1, 2, 3, 4, 5, 6}, $n(s) = 6 = 6^1$.
Die is thrown twice or two dice are thrown simultaneously	S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (6,6)}, $n(s) = 36 = 6^2$.

- **Event:** Collection of some including no outcome or all outcomes from the sample space.
- **Probability of an event:**

$$P(E) = \frac{\text{no of outcomes favourable to the event}}{\text{Total no. of outcomes in the sample space}}$$

$$= \frac{n(E)}{n(S)}$$

- **Sure Event:** If no. of outcomes favourable to the event is equal to no. of total outcomes of the sample space or an event whose probability is 1.
- **Impossible Event:** Having no outcome or an event whose probability is 0.
- **Range of Probability:** Probability of an event always lies between 0 and 1 (0 and 1 inclusive) i.e. $0 \leq P(E) \leq 1$.
- **Complementary Event:** Event which occurs only when E does not occur and denoted by $\overline{E}$.
  Probability of a complementary Event
  $P(\overline{E}) = 1 - P(E)$
- **Sum of Probabilities:** Sum of all the probabilities is 1 i.e $P(E_1) + P_1(E_2) + P(E_3) - +P(E_n) = 1$ and $P(E) + P(\overline{E}) = 1$.

CHECK YOUR PROGRESS:

1. A die is thrown once. The probability of getting a prime number is :

   (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $\frac{1}{6}$

2. Two coins are tossed once. The Probability of getting atleast one head is :

   (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1

3. A card is drawn from a pack of 52 cards. The probability that it is a face card, is :

   (A) $\frac{4}{13}$ (B) $\frac{3}{13}$ (C) $\frac{2}{13}$ (D) $\frac{1}{13}$

4. A pair of dice is thrown once. The probability of having a sum 11 on the two dice is :

   (A) $\frac{1}{36}$ (B) $\frac{1}{12}$ (C) $\frac{1}{18}$ (D) $\frac{1}{9}$

5. Which of the following cannot be the probability of an event-

   (A) $\frac{2}{3}$ (B) 15% (C) 0.7 (D) 1.5

6. A coin is thrown twice. Find the probability of getting one head.
7. A die is thrown once. Find the probability of getting an even number.
8. A card is drawn from a well-shuffled deck of 52 playing cards. Find the probability that it is not an ace.

STRETCH YOURSELF

1. Cards marked with numbers 3, 4, 5 ... 19 are kept in a box and mixed thoroughly. If one card is drawn at random from the box, find the probability of getting.

   (i) A prime number (ii) A perfect square
2. A bag contains 12 balls out of which x are white. If 6 more white balls are put in the bag, the probability of getting a white ball becomes double. Find the value of x.
3. Find the probability of getting 53 Sundays in a non leap year.
4. If a number x is chosen from the numbers 1, 2, 3 and a number y is selected from the numbers 1, 4, 9, then find $P(xy < 9)$.

ANSWERS

CHECK YOUR PROGRESS :

1. A 2. C 3. B 4. C

5. D 6. $\frac{1}{2}$ 7. $\frac{1}{2}$ 8. $\frac{12}{13}$

STRETCH YOURSELF :

1. (i) $\frac{7}{17}$ (ii) $\frac{3}{17}$
2. 3
3. $\frac{1}{7}$
4. $\frac{5}{9}$

PRACTICAL ACTIVITIES

You have learnt many concepts of Mathematics in Book-I and Book-II of NIOS Secondary Course. Some of the concepts in maths are of abstract nature and learning such concepts become easier when learnt through activities performed in Mathematics Laboratory. Performing activities of mathematics not only improve the problem solving skills but also makes concrete understanding of mathematical concepts. Mathematics relies on both logic and creativity, and it is pursued both for variety of practical purposes and for your intrinsic interest. In mathematics the concepts for which the proof/verification is done by experimentation or activities are better understood by learners. For developing problem solving skill and concrete understanding of mathematical concepts, NIOS has developed a practical manual, which is in addition to two books of mathematics.

There are thirty (30) activities on different concepts in the practical manual. In order to prepare activities on these concepts, different materials as well as principle requires for the preparation. The details of the preparation for the activities are in the practical manual.

As 15 marks are allotted for the practical examination. You have to prepare yourself for practical examination as per following guidelines.

Time allowed: 2½ hours *Maximum Marks: 15*

1. **Distribution of Marks:**

S.No.	**Activities**	**Marks Distribution**
(i)	Assessment of activity performed: (Two activities out of given three activities)	2 X 4 Marks = 08 Marks
(ii)	Record Note book of activities: (At least Five Activities From each of the section)	= 03 Marks
(iii)	Viva-voce, based on the activities:	= 04 Marks
		Total = 15 Marks

2. **List of Activities:**

From the given list of activities in sections A, B and C, one activity from each section may be given to the learners. Of these three activities, the learner will have to choose any two activities and perform them at practical examination centre.

2.1 Section-A (Algebra)

1. Verification of the Identity $(a+b)^2 = a^2+2ab+b^2$
2. Verification of the identity $(a-b)^2 = a^2-2ab+b^2$
3. Verification of the identity $(a^2-b^{2)} = (a+b)\ (a-b)$
4. Verification of the identity $(a+b)^3 = a^3+ 3ab^2+3ab^2+b^3$
5. To verify the identity $a^3-b^3 = (a-b)\ (a^2+ab+b^2)$
6. To find H.C.F of two given natural numbers by division method.
7. Demonstration of the concept of Equivalent Fractions.
8. To verify that a linear equation in two variables has infinite number of solutions.
9. To find the condition for consistency of a system of linear equations in two variables.
10. To verify the relation between roots and coefficients of a quadratic equation.
11. To verify graphically that a quadratic polynomial can have at most two zeroes.
12. To verify that a given sequence is an A.P.
13. To find the sum of first n odd natural numbers.
14. To find the sum of first n natural numbers.
15. To find the sum of first n terms of an Arithmetic Progression.

2.2 Section-B (Geometry)

1. To verify that the sum of the angles of a triangle is 180^0
2. To verify that the angles opposite to equal sides of a triangle are equal.
3. To verify the midpoint theorem.
4. To verify basic proportionality theorem.
5. To verify Pythagoras theorem.
6. To verify the relation between the ratio of areas of two similar triangles and ratio their sides
7. To find the area of a circle.
8. To demonstrate that the opposite angles of a cyclic quadrilateral are supplementary.
9. To verify that equal chords of congruent circles subtend equal angles at the centre of the respective circles.

2.3 Section- C (Mensuration)

1. To find the area of a trapezium.
2. To find the total surface area of a cube.
3. To find the formula for the curved surface area of a cone.

4. To find the relationship among the volumes of a right circular cylinder, right circular cone and a hemisphere of same radius and same heights.
5. To draw a triangle equal in area to a parallelogram.
6. To find the in centre of different types of triangles.

3. Materials Required:

- Sheets of paper of different colors
- Wooden boards
- Threads
- Nails, Pins and Clips
- Thermocole sheets.
- Cardboards square and triangular grids.
- Wooden and paper strips.
- Paper cutter
- Pair of scissors
- Adhesive/Fevicol
- Sketch pens
- Geometry boxes (small and bigger both)
- Graph papers (inches/cm both)
- Pencils of different colors
- Sketch pens/coloured ball point pen and markers
- Color box
- Knobs
- Tracing papers
- Acrylic sheets
- Eraser and Sharpner
- String
- Stand with grooves so that it can keep any rod, fixed on it through pulleys
- Screws and screw driver
- Plastic sheets, plastic balls, sand
- Cello tape

Note: Material for practical examination will be provided by the centre superintendant at the practical examination centre as per requirement of the activities.

SAMPLE QUESTION PAPER

Subject: Mathematics (211)
Secondary Course

Mark: 85 **Time: 2 ½ Hrs.**

Note:

1. Question Numbers 1-15 are multiple choice questions. Each question carries one mark
2. Question Numbers 16-25 carry 2 marks each
3. Question Numbers 26-33 carry 4 marks each
4. Question Numbers 34-36 are of 6 marks each
5. **All** questions are compulsory

1. In the word PERCENTAGE, what percent of the letters are E's? (1)

(A) 10% (B) 20% (C) 30% (D) 40%

2. There are 500 students in a school. If the number of boys in the school is 300, then the percentage of the girls is: (1)

(A) 20% (B) 40% (C) 60% (D) $66\frac{2}{3}\%$

3. Two figures are called congruent figures if they have: (1)

A. Same size and different shape
B. Same size and same shape
C. Different size and same shape
D. Different size and different shape

4. $(\sin A - \cos A)^2 + 2\sin A\cos A$ equal to : (1)

(A) 1 (B) $4\sin A\cos A$ (C) $1+4\sin A\cos A$ (D) $1-4\sin A\cos A$

5. If $\sin\theta = \frac{\sqrt{3}}{2}$ then the value of θ is: (1)

(A) 30º (B) 90º (C) 60º (D) 0º

6. $\left(\sqrt{3}+\sqrt{7}\right)\left(\sqrt{3}-\sqrt{7}\right)$ is equal to : (1)

(A) 4 (B) -4 (C) 2 (D) -2

7. The roots of the equation $x^2 - 18x + 81 = 0$ are : (1)

(A) 9, 9 (B) 9, 0 (C) 9, -9 (D) -9, -9

8. If $\sin\alpha = \frac{11}{15}$, then value of $\cos\alpha$ is: (1)

(A) $\frac{15\sqrt{26}}{2}$ (B) $\frac{15}{11}$ (C) $\frac{2\sqrt{26}}{15}$ (D) $\frac{15}{2\sqrt{26}}$

9. If $\sqrt{3}\tan\theta = 3\sin\theta$, then the value of $\cos\theta$ is: (1)

(A) $\frac{1}{3}$ (B) 3 (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{3}$

10. The arithmetic mean of first five natural numbers is: (1)

(A) 3 (B) 4 (C) 5 (D) 2

11. The mode of a set of observations : (1)

(A) is the value which occurs most frequently

(B) is the central value

(C) is the sum of observations

(D) divides observations into two equal parts

12. Two adjacent angles of a rhombus are in the ratio 4: 5. The measures of adjacent angles are. (1)

(A) 60^0, 90^0

(B) 80^0, 100^0

(C) 70^0, 110^0

(D) 60^0

13. Which of the following are right triangles? (1)

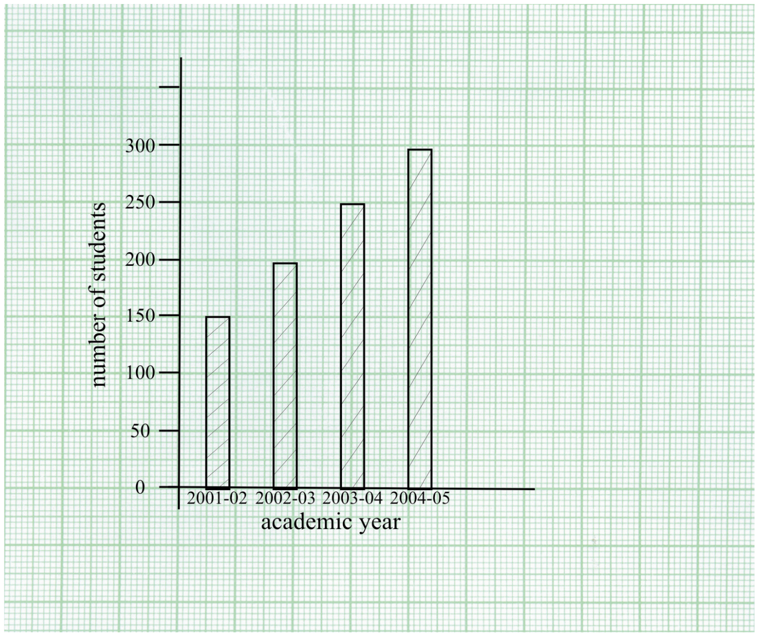
(A) AB = 5 cm, BC= 12 cm, CA = 13 cm

(B) AB = 8 cm, BC = 6 cm, CA = 10 cm

(C) AB = 24 cm, BC = 25cm, CA = 7 cm

(D) all of the above

14. In the given bar graph, the numbers of students in 2003-2004 are (1)



(A) 150 (B) 200 (C) 250 (D) 300

15. For drawing a frequency polygon for a grouped frequency distribution, the points to be plotted with ordinate as frequency and abscissa as (1)

(A) Lower limit (B) upper limit

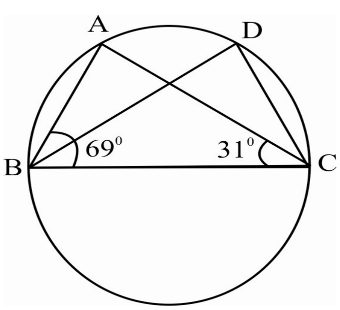
(C) class marks (D) any value of the class

16. Find two rational numbers between $\frac{5}{6}$ and $\frac{7}{8}$ (2)

17. Factorise: $3x^2 - 2x - 5$ (2)

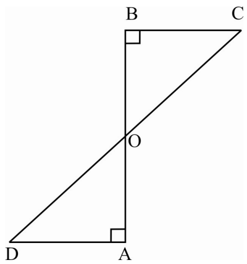
18. If PA and PB are tangents from an outside point P such that PA = 10 cm and $\angle APB = 60^0$, find the length of chord AB. (2)

19. In figure $\angle ABC = 69^0$ $\angle ACB = 31^0$ find $\angle BDC$.



20. If $\tan\theta = \frac{4}{3}$, find the value of $\frac{3\sin\theta - 2\cos\theta}{3\sin\theta + 2\cos\theta}$ (2)

21. In given figure AD and BC are perpendiculars to a line segment AB. Show that CD bisects AB. (2)



22. A man goes 10 m due east and then 24 m due north. Find the distance of the man from the starting point. (2)

23. The circumference of the base of a right circular cone is 88cm and its height is 10cm. Find the volume of the cone. (2)

24. Cost price of 23 articles is equal to the selling price of 20 articles. Find the loss or gain percent. (2)

25. Volume of a cylinder is 252 cm^3 and its height is 7cm. Find the curved surface area of the cylinder. (2)

26. If $x = 4 + \sqrt{15}$, find the value of $x + \frac{1}{x}$ and $x^2 + \frac{1}{x^2}$ (4)

27. A computer is available for Rs. 34000 cash or Rs. 20000 cash down payment followed by five equal monthly instalments of Rs. 3000 each. Find the rate of interest per annum charged under the instalment plan. (4)

28. Draw a circle of radius 3 cm. Take a point P at a distance of 5.5 cm from the centre of the circle. From point P, draw two tangents to the circle. (4)

29. D, E, F are the mid points of sides BC, CA and AB of Δ ABC. Show that AD bisects EF. (4)

30. If $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$, then show that $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$. (4)

31. If the mean of the following distribution is 6, find the value f p. (4)

x_i:	2	4	6	10	p+5
f_i:	3	2	3	1	2

32. Two dice are thrown simultaneously. Find the probability of getting (4)

i. Same number on both dice.

ii. A total of 10.

33. Simplify and express the result in the lowest terms: (4)

$$\frac{\left(x^2 - 7x + 12\right)\times\left(x^2 - 2x - 24\right)}{\left(x^2 - 2x - 3\right)\times\left(x^2 - 16\right)}$$

34. The sum of a two digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there? (6)

35. The radius and height of a right circular cone are 3 cm and 4 cm respectively. Find its (6)

(i) Curved surface area

(ii) Total surface area

(iii) Volume

36. Prove that triangles on the same base and between the same parallels are equal in area (6)

ANSWER OF SAMPLE QUESTION PAPER

1.	C	2.	B	3.	B	4.	A
5.	C	6.	B	7.	A	8.	C
9.	C	10.	A	11.	A	12.	B
13.	D	14.	C	15.	C		

16. $\frac{41}{48}$ and $\frac{81}{96}$ are two rational numbers

17. $(3x-5)\ (x+1)$

18.	AB = 10 cm	19.	$\angle BDC = 80^0$	20.	1/3	22.	26 m
23.	6160 cm^3	24.	15%	25.	264 cm^2	26.	8 and 62
27.	30%	31.	P= 7	32.	1/6 and 1/12		

33. $\frac{x-6}{x+1}$

34. Then number is 42 or 24 and there are two numbers

35. (i) $\frac{330}{7} = 47.14 cm^2$; (ii) $\frac{22\times3\times8}{7} = \frac{528}{7} = 75.42\,cm^2$; (iii) $\frac{264}{7} = 37.71\ cm^3$